BEAL MOCK

Edexcel GCE

Further Core Mathematics

Advanced Subsidiary

September 2020

Time: 1 hour 40 minutes

Calculators may be used in this examination.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (AS FM Core Mathematics FMCM 1), the paper reference (6663), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

The marks for the parts of questions are shown in round brackets, e.g. (2).

There are 10 questions in this question paper. The total mark for this paper is 80.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner.

Answers without working may not gain full credit.

TURN OVER

1 Use standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n} r(r-2)(r+2) = \frac{1}{4}n(n+1)(an^2+bn+c)$$

where a, b and c are integers to be found. **(4)**

(Total 4 marks)

---

2 The cubic equation

$$2z^3 - z^2 + z = 0$$

has three roots α, β and γ.

Without solving the equation, find the cubic equation whose roots are $(\alpha + 1)$, $(\beta + 1)$ and $(\gamma + 1)$, giving your answer in the form $w^3 + aw^2 + bw + c = 0$, where a, b and c are constants to be found. **(4)**

(Total 4 marks)

---

3 Three planes are defined by the system of equations

$$4x + y + 2z = 4$$

$$kx - 2y + 3z = 10$$

$$y + (k - 1)z = 2$$

where k is a constant.

(a) Find the values of k for which the three planes do not intersect at a unique point. **(3)**

For both of the values of k found in part (b),

(b) (i) show that the equations form an inconsistent system. **(3)**

(ii) Hence, identify the geometrical configuration of the planes in these cases. **(1)**

(Total 7 marks)

---

4 The quadratic equation

$$3x^2 - kx + 14 = 0$$

has two positive real roots α and β in the ratio 7:6.

Find the value of the constant k. **(5)**

(Total 5 marks)

---

5 The complex numbers z_1 and z_2 are such that

$$z_1 = -8\sqrt{3} + 4\text{i}, \qquad z_2 = 7 + \text{i}\sqrt{3}$$

(a) Find

(i) $z_1 z_2$ **(2)**

(ii) $\dfrac{z_1}{z_2}$ **(3)**

giving your answers in the form $x + \text{i}y$.

On an Argand diagram with origin O, the complex numbers z_1 and z_2 are represented by the points A and B respectively.

(b) Show the points A and B on an Argand diagram. **(1)**

(c) Without using a calculator, prove that the angle OAB is $\dfrac{5\pi}{6}$. **(2)**

(Total 8 marks)

TURN OVER

6 The 2×2 matrix **A** represents a reflection in the line $y = x$.

(a) Write down the matrix **A**. **(1)**

The transformation T_1 is a reflection in the line $y = x$ followed by an enlargement about the origin by a scale factor k, where $k \neq 0$ and $k \neq 1$. The 2×2 matrix **B** represents T_1.

(b) Find, in terms in k, the matrix **B**. **(3)**

The point P is the only point invariant under T_1.

(c) Write down the coordinates of P. **(1)**

The transformation T_2 is represented by the matrix **C**, where

$$\mathbf{C} = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}$$

The transformation T is the transformation T_1 followed by the transformation T_2.

The point $(1, 4)$ is mapped to the point $(20, 32)$ by the transformation T.

(d) Find the image of the point $(2, 3)$ under T. **(5)**

(e) Is the point P also invariant under T? Justify your answer. **(2)**

(Total 12 marks)

7

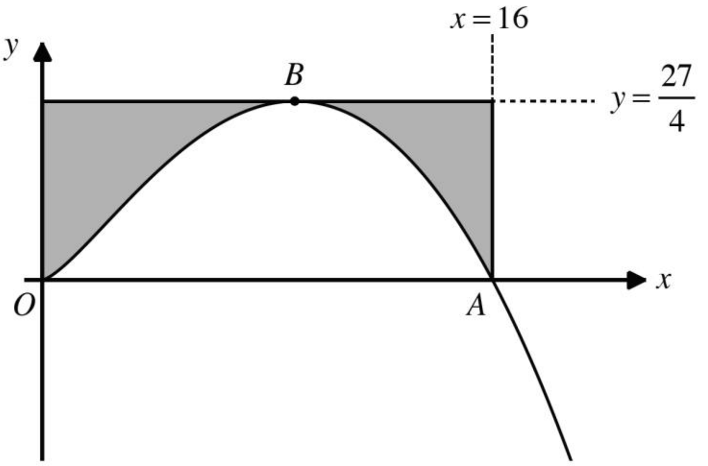

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \sqrt{x^3} - \frac{1}{4}x^2$$

The curve C crosses the x-axis at the origin O and the point $A(a, 0)$.

(a) Show that $a = 16$. **(2)**

The maximum point on the curve C occurs at $B(b, c)$.

(b) Find the value of b and hence show that $c = \frac{27}{4}$. **(4)**

The shaded region R, shown shaded in Figure 1, is bounded by the curve, the y-axis and the lines $y = \frac{27}{4}$ and $x = 16$. The region R is rotated 360 degrees about the x-axis to form a solid with volume V.

(c) Use calculus to find V.
Give your answer to three significant figures. **(7)**

(Total 13 marks)

TURN OVER

8 The lines $\mathbf{r}_1$ and $\mathbf{r}_2$ are defined such that

$$\mathbf{r}_1 = (\mathbf{i} + \mathbf{j}) + \lambda(-2\mathbf{i} + \mathbf{j} + 3\mathbf{z})$$

$$\mathbf{r}_2 = (-2\mathbf{i} + \mathbf{z}) + \mu(2\mathbf{i} + \mathbf{j} + \mathbf{z})$$

(a) Show that the lines $\mathbf{r}_1$ and $\mathbf{r}_2$ are skew. **(4)**

(b) Show that the vector $(\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ is normal to both $\mathbf{r}_1$ and $\mathbf{r}_2$. **(3)**

The plane Π is equidistant from the lines $\mathbf{r}_1$ and $\mathbf{r}_2$.

(c) Find the equation of the plane Π.
Give your answer in the form $ax + by + cz = d$. **(3)**

(Total 10 marks)

9

$$f(z) = z^3 - 13z^2 + pz + q$$

where p and q are constants.

The equation $f(z) = 0$ has roots z_1, z_2 and z_3.
When plotted on an Argand diagram, the points representing z_1, z_2 and z_3 lie on the circle $|z| = 5$.

Given that $z_1 = 5$, find the values of p and q. **(8)**

(Total 8 marks)

---

10 (a) Prove by induction that for $n \in \mathbb{Z}^+$

$$f(n) = n(n + 1)$$

is divisible by 2. **(4)**

(b) Hence, or otherwise, use induction to prove that for $n \in \mathbb{Z}^+$

$$g(n) = n^3 - n$$

is divisible by 6. **(5)**

(Total 9 marks)
