3 $\dfrac{2x^2+2x-18}{x(x-3)^2} \equiv \dfrac{P}{x}+\dfrac{Q}{x-3}+\dfrac{R}{(x-3)^2}$

$$\equiv \frac{P(x-3)^2+Qx(x-3)+Rx}{x(x-3)^2}$$

$$2x^2+2x-18 \equiv P(x-3)^2+Qx(x-3)+Rx$$

Let $x=0$:

$-18 = P\times(-3)^2+0+0$

$-18 = 9P$

$P=-2$

Let $x=3$:

$18+6-18 = 0+0+R\times 3$

$6 = 3R$

$R=2$

Equating terms in x^2:

$2 = P+Q$

$2 = -2+Q$

$Q=4$

$P=-2,\ Q=4,\ R=2$

6 Using algebraic long division:

$$\begin{array}{r} 4x-13 \\ x^2+2x-1\overline{)\,4x^3-5x^2+3x-14} \\ \underline{4x^3+8x^2-4x} \\ -13x^2+7x-14 \\ \underline{-13x^2-26x+13} \\ 33x-27 \end{array}$$

$$\frac{4x^3-5x^2+3x-14}{x^2+2x-1} \equiv 4x-13+\frac{33x-27}{x^2+2x-1}$$

So $A=4$, $B=-13$, $C=33$ and $D=-27$.

7 b Coefficient of x^3 is

$$\frac{n(n-1)(n-2)a^3}{3!} = \frac{(-2)\times(-3)\times(-4)\times 3^3}{3\times 2\times 1} = -108$$

c $(1+3x)^{-2}$ is valid if $|3x|<1 \Rightarrow |x|<\frac{1}{3}$

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14 a $\frac{12x+5}{(1+4x)^2} \equiv \frac{A}{1+4x} + \frac{B}{(1+4x)^2}$

$$\equiv \frac{A(1+4x)+B}{(1+4x)^2}$$

$$12x+5 \equiv A(1+4x)+B$$

Let $x=-\frac{1}{4}$:

$-3+5=0+B$

$B=2$

Let $x=0$:

$5=A\times 1+B$

$5=A+2$

$A=3$

$A=3, B=2$

14 b $\frac{12x+5}{(1+4x)^2} \equiv \frac{3}{1+4x} + \frac{2}{(1+4x)^2}$

$$=3(1+4x)^{-1}+2(1+4x)^{-2}$$

$$3(1+4x)^{-1} = 3\left(1+(-1)(4x)+\frac{(-1)(-2)}{2!}(4x)^2+\ldots\right)$$

$$=3\left(1-4x+16x^2+\ldots\right)$$

$$=3-12x+48x^2+\ldots$$

$$2(1+4x)^{-2} = 2\left(1+(-2)(4x)+\frac{(-2)(-3)}{2!}(4x)^2+\ldots\right)$$

$$=2\left(1-8x+48x^2+\ldots\right)$$

$$=2-16x+96x^2+\ldots$$

$$\frac{12x+5}{(1+4x)^2} = 3-12x+48x^2+2-16x+96x^2+\ldots$$

$$=5-28x+144x^2+\ldots$$