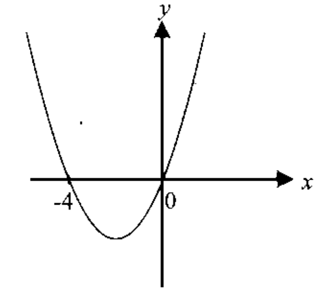
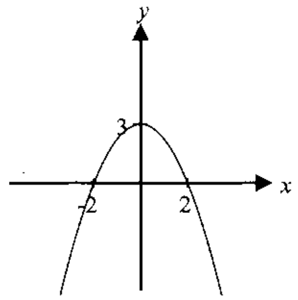
17 Area of circle = πr^2
Area of entire circle = $\pi \times 11^2 = 121\pi$
Area of inner two circles (grey and white) = $\pi \times (5 + 2)^2 = 49\pi$
Area of innermost grey circle = $\pi \times 5^2 = 25\pi$
Shaded area of entire circle = $121\pi - 49\pi + 25\pi = 97\pi$
Area of shaded sector = $\frac{120}{360} \times 97\pi = \frac{1}{3} \times 97\pi = \frac{97}{3}\pi$
[4 marks available — 1 mark for finding the area of the entire circle, 1 mark for finding the area of one of the inner circles, 1 mark for the shaded area of the entire circle, 1 mark for the correct answer]

18 a)



[2 marks available — 1 mark for correct intercepts (–4, 0) and (0, 0), 1 mark for correctly drawn graph]

b)



[2 marks available — 1 mark for correct intercepts (–2, 0), (2, 0) and (0, 3), 1 mark for correctly drawn graph]

19 $\sin 60° = \frac{\sqrt{3}}{2}$ ***[1 mark]*** $\tan 30° = \frac{1}{\sqrt{3}}$ ***[1 mark]***
$2 \times \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{2\sqrt{3}}{2\sqrt{3}} = 1$ ***[1 mark]***
[3 marks available in total — as above]

20 a) $-\frac{6}{2} = -3$, so $a = -3$, $(x - 3)^2 = x^2 - 6x + 9$
so $b = -11$ and $x^2 - 6x - 2 = (x - 3)^2 - 11$
[2 marks available — 1 mark for the correct value of a, 1 mark for the correct value of b]

b) $(3, -11)$ ***[1 mark]***

21 To prove all triangles are similar, show that the angles are the same in each triangle. Let angle $ABC = x$, then
Triangle ABC contains:
Angle $BCA = 90°$ (as the angle in a semicircle is 90°)
Angle $ABC = x$, and angle $BAC = 180° - 90° - x = 90° - x$
Triangle ABD contains:
Angle $DAB = 90°$ (as angle where a tangent meets a radius is 90°)
Angle ABD = angle $ABC = x$ and
Angle $ADB = 180° - 90° - x = 90° - x$
Triangle ACD contains:
Angle $ACD = 180°$ – angle $BCA = 180° - 90° = 90°$
Angle ADC = angle $ADB = 90° - x$
Angle $DAC = 180° - 90° - (90° - x) = x$
The three angles in all triangles are 90°, x and $(90° - x)$ so all three triangles are similar.
[4 marks available — 1 mark for finding the angles in triangle ABC, 1 mark for finding the angles in triangle ABD, 1 mark for finding the angles in triangle ACD, 1 mark for a conclusion that all triangles contain same angles so are similar]

22 $c + 3 : a + 3 = 2 : 3$
$\frac{c+3}{a+3} = \frac{2}{3}$, so $3(c + 3) = 2(a + 3)$
$3c + 9 = 2a + 6$
$3c - 2a = -3$ ***[1 mark]***

$c + 1 : a + 5 = 1 : 2$
$\frac{c+1}{a+5} = \frac{1}{2}$, so $2(c + 1) = a + 5$
$2c + 2 = a + 5$
$2c - a = 3$ ***[1 mark]***

$2c - a = 3 \xrightarrow{\times 2} 4c - 2a = 6$ ***[1 mark]***

$3c - 2a = -3$
$-\ \ 4c - 2a = 6$
$-c = -9$
$c = 9$ ***[1 mark]***

$2c - a = 3$
$2 \times 9 - a = 3$
$18 - a = 3$
$a = 15$ ***[1 mark]***

The ratio $c : a = 9 : 15 = 3 : 5$ in its simplest terms. ***[1 mark]***
[6 marks available in total — as above]

Set 2 Paper 2 — Calculator

1 $7 \times 4 \times 5 = 140$ choices ***[1 mark]***
$140 \times £3.50 = £490$ ***[1 mark]***
[2 marks available in total — as above]

2 1 fathom = 1.8 m
1.782 km = 1.782 × 1000 = 1782 m ***[1 mark]***
1782 m = 1782 ÷ 1.8 = 990 fathoms ***[1 mark]***
[2 marks available in total — as above]

3 a) E.g.
The number of people using the site grew slowly between 2007 and 2009, then increased at a steady rate between 2009 and 2012. ***[1 mark]***
The number of people increased more slowly between 2012 and 2014. ***[1 mark]***
[2 marks available in total — as above]

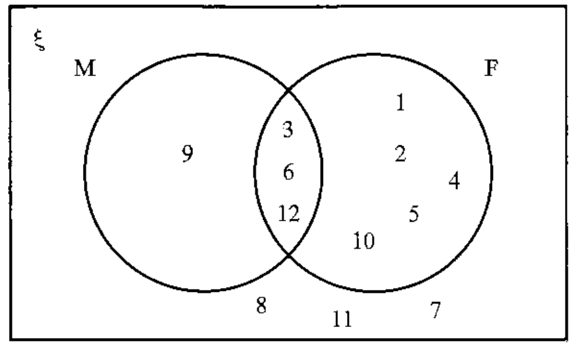
b) E.g. The graph suggests there could be 125 000 people using the website by 2017. This is unreliable because the trend may change as 2017 is outside the data that is available, and there may be a sharp increase or decrease in the number of users.
[2 marks available — 1 mark for a prediction between 120 000 to 130 000, 1 mark for a correct point on reliability]
You could argue that the graph is reliable as it shows a smooth curve which may continue past the data points.

4 The values in the two ratios that represent 'milk' are 3 and 7. 21 is the lowest common multiple of 3 and 7, so multiply the first ratio by 7 to get 14 : 21, and the second ratio by 3 to get 21 : 6 ***[1 mark]***
So, the ratio of all three chocolates is 14 : 21 : 6 ***[1 mark]***
There are 14 + 21 + 6 = 41 parts,
So one part is 123 ÷ 41 = 3 chocolates
There are 14 × 3 = 42 plain chocolates ***[1 mark]***
[3 marks available in total — as above]

5 a) The difference between each term is 7, so the expression for the n^{th} term will include a $7n$. ***[1 mark]***
The sequence $7n$ is: 7, 14, 21, 28...
Each term in the given arithmetic sequence is 4 less than this sequence, so the n^{th} term = $7n - 4$ ***[1 mark]***
[2 marks available in total — as above]

b) If 1024 was in the sequence then $7n - 4 = 1024$ for some integer n. Rearrange to get $7n = 1028$ ***[1 mark]***, then $n = 146.8571...$
n is not an integer, so 1028 is not a term of the sequence.
[1 mark]
[2 marks available in total — as above]

6 a)



[3 marks available — 1 mark for the correct values in the circles, 1 mark for the correct values in the overlap, 1 mark for correct values outside of the circles]

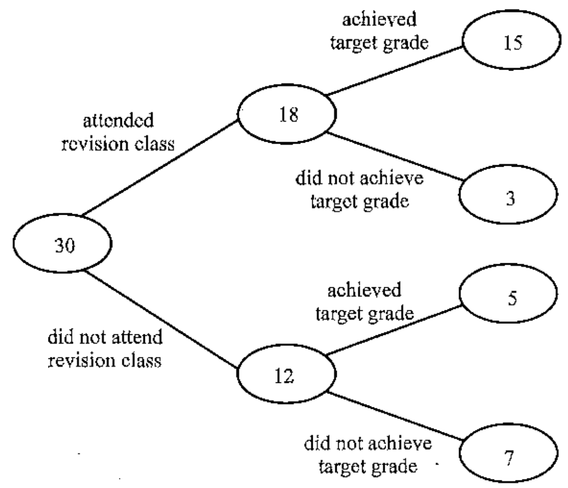
b) M ∩ F = {3, 6, 12}. There are 12 numbers in total,
so P(M ∩ F) = $\frac{3}{12}$ (or $\frac{1}{4}$). ***[1 mark]***
(M ∪ F)′ = {7, 8, 11} so P(M ∪ F)′ = $\frac{3}{12}$ (or $\frac{1}{4}$). ***[1 mark]***
[2 marks available in total — as above]
If you've stated that the probability is equal as the set M ∩ F has the same number of elements as the set (M ∪ F)′ you'll still get the marks.

7 a) $g(-2) = (-2)^2 - 3 = 4 - 3 = 1$ ***[1 mark]***

b) Set $x = 3y - 5$, and rearrange to make y the subject.
$3y = x + 5$
$y = \frac{x+5}{3}$
So $f^{-1}(x) = \frac{x+5}{3}$
[2 marks available — 1 mark for the correct method, 1 mark for the correct answer]

c) $gf(x) = g(3x - 5)$
$g(3x - 5) = (3x - 5)^2 - 3$ ***[1 mark]***
$= 9x^2 - 30x + 25 - 3 = 9x^2 - 30x + 22$ ***[1 mark]***
[2 marks available in total — as above]

8 a) 60% of 30 = 0.6 × 30 = 18 attended revision class.
30 – 18 = 12 did not attend revision class.
18 – 15 = 3 attended but did not achieve target grade.
$30 \times \frac{2}{3} = 20$ in total achieved target grade.
20 – 15 = 5 did not attend but achieved target grade.
12 – 5 = 7 did not attend and did not achieve target grade.



[3 marks available — 3 marks for tree fully correct, otherwise 2 marks if one value is incorrect, or 1 mark if two values are incorrect]
Check you haven't made any mistakes by checking each 'column' of numbers adds to the same number (30 in this case).

b) There are 12 students who didn't go to revision class and 5 achieved their target grade so the probability is $\frac{5}{12}$.
[1 mark]

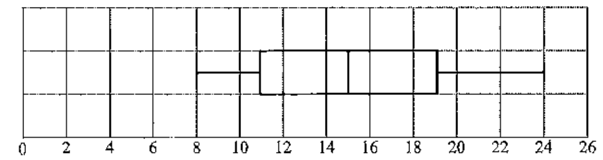
9 $c \propto a^2$, so $c = ka^2$.
When $a = 10$, $c = 1500$ so $1500 = k \times 10^2 = 100k$.
So, $k = 15$ and $c = 15a^2$ ***[1 mark]***
When $c = 2940$, $2940 = 15a^2$
$a^2 = 196$ ***[1 mark]***
$a = 14$ ***[1 mark]***
[3 marks available in total — as above]

10 d is the exterior angle of a regular pentagon, so
$d = 360° \div 5 = 72°$. ***[1 mark]***
The triangle is isosceles, so the unmarked angle is also 72°.
$c = 180° - 72° - 72° = 36°$ ***[1 mark]***
The ratio $c : d$ is 36 : 72 which simplifies to 1 : 2. ***[1 mark]***
[3 marks available in total — as above]

11 a) 30 cm = 0.3 m
$T = 2\pi \times \sqrt{\frac{0.3}{9.78}} = 1.1004... = 1.10$ (3 s.f) seconds
[2 marks available — 1 mark for using 0.3 in the formula, 1 mark for the correct answer]
Make sure that you check all measurements have the correct units before putting them into a formula.

b) The change in T from the Equator to the North Pole is
$2\pi \times \sqrt{\frac{0.3}{9.78}} - 2\pi \times \sqrt{\frac{0.3}{9.832}} = 0.002913...$ seconds
The percentage change is
$\frac{0.002913...}{1.1004...} \times 100 = 0.2647...\% = 0.265\ \%$ (3 s.f.)
[3 marks available — 1 mark for finding the difference between the two periods, 1 mark for using the percentage change formula, 1 mark for the correct answer]

12 a) Minimum = 8, maximum = 24
Median = (11 + 1) ÷ 2 = 6^{th} value = 15
Lower quartile = (11 + 1) ÷ 4 = 3^{rd} value = 11
Upper quartile = 3(11 + 1) ÷ 4 = 9^{th} value = 19



[3 marks available — 1 mark for correctly plotting the minimum and maximum values, 1 mark for correctly plotting the lower and upper quartiles, 1 mark for correctly plotting the median]

b) E.g. On average the students scored higher marks in the chemistry exam than the physics exam, as the median is higher for chemistry. The students' marks on the chemistry exam were more spread out (inconsistent) as the range and interquartile range are both larger.
[2 marks available — 1 mark for comparing the average marks with a correct conclusion, 1 mark for comparing the spread of marks with a correct conclusion]

13 The gradient of the line from (2, 7) to (5, 13) = $\frac{13-7}{5-2} = \frac{6}{3} = 2$
[1 mark]
Two lines are perpendicular if their gradients multiply to equal –1, so a perpendicular line to 2 will have a gradient of $-\frac{1}{2}$.
[1 mark]
The equation $2y = 13 - x$ can be rearranged to give $y = \frac{13}{2} - \frac{1}{2}x$.
The gradient is $-\frac{1}{2}$ and so the two lines are perpendicular.
[1 mark]
[3 marks available in total — as above]

14 If x is the multiplier for the annual percentage increase, then $750 \times x^4 = 844.13$.

$x^4 = \frac{844.13}{750} = 1.1255...$ ***[1 mark]***

$x = \sqrt[4]{1.1255...} = 1.0299... = 1.03$ (3 s.f.) ***[1 mark]***

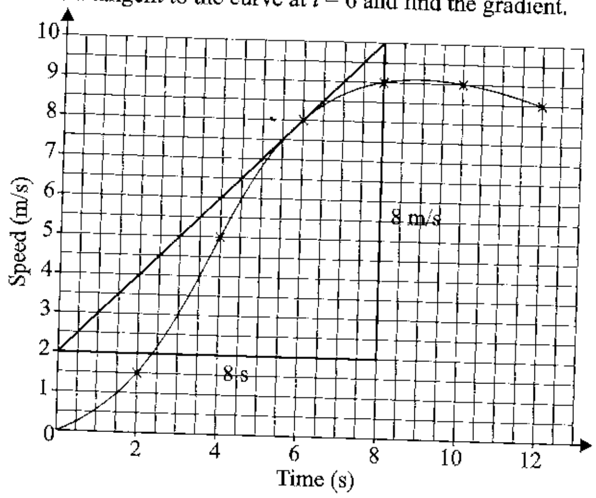
So the annual rate of interest was 3%. ***[1 mark]***

[3 marks available in total — as above]

15 a) The estimated 32 000 is 107% of the original value, so
32 000 ÷ 107 × 100 = 29 906.5420... = 29 907 ***[1 mark]***
and 32 000 – 29 907 = 2093 ***[1 mark]***
[2 marks available in total — as above]

b) 32 000 to the nearest 1000 means the minimum population was 31 500 and the maximum population was 32 499.
31 500 ÷ 107 × 100 = 29 439.2523... = 29 439
31 500 – 29 439 = 2061 ***[1 mark]***
32 499 ÷ 107 × 100 = 30 372.897... = 30 373
32 499 – 30 373 = 2126 ***[1 mark]***
2093 – 2061 = 32 and 2126 – 2093 = 33. The estimate for part (a) could be a maximum of 33 away from the actual value, not 500, so Mikko is incorrect. ***[1 mark]***
[3 marks available in total — as above]

16 a) Draw a tangent to the curve at $t = 6$ and find the gradient.

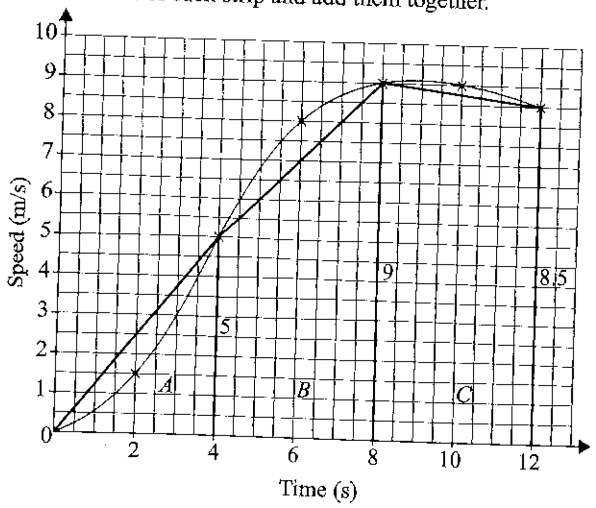


Gradient = acceleration = $\frac{10-2}{8-0} = \frac{8}{8} = 1 \text{ m/s}^2$

[2 marks available — 1 mark for drawing a tangent, 1 mark for an answer in the range 0.8 – 1.2 m/s²]

The accuracy of your tangent will affect the answer you get for the acceleration — award yourself full marks if you're in the range above.

b) Draw 3 equal strips of width 4, and label them A, B and C. Find the area of each strip and add them together.



$A = \frac{1}{2} \times 4 \times 5 = 10$ m

$B = \frac{1}{2}(5 + 9) \times 4 = 28$ m

$C = \frac{1}{2}(9 + 8.5) \times 4 = 35$ m

Total distance = 10 + 28 + 35 = 73 m ***[1 mark]***

[3 marks available — 1 mark for drawing 3 strips of width 4 seconds, 1 mark for the correct area of two or more strips, 1 mark for the correct answer]

17 Find the prime factors:
$450 = 2 \times 3^2 \times 5^2$, so $450^3 = (2 \times 3^2 \times 5^2)^3 = 2^3 \times 3^6 \times 5^6$
$240 = 2^4 \times 3 \times 5$, so $240^3 = (2^4 \times 3 \times 5)^3 = 2^{12} \times 3^3 \times 5^3$
Multiply the prime factors that appear in either number:
$\text{LCM} = 2^{12} \times 3^6 \times 5^6 = (2^2 \times 3 \times 5)^6 = 60^6$
[4 marks available — 1 mark for finding prime factors of 450, 1 mark for finding prime factors of 240, 1 mark for finding the LCM of 450³ and 240³, 1 mark for showing the LCM = 60⁶]
You can use factor trees to find the prime factors of each number.

18 $6x - 3$ factorises to $3(2x - 1)$ and
$2x^2 + 7x - 4$ factorises to $(x + 4)(2x - 1)$ ***[1 mark]***
$x^2 - 16 = (x - 4)(x + 4)$ ***[1 mark]***

So, $\frac{6x-3}{2x^2+7x-4} \div \frac{15}{x^2-16}$

$= \frac{3(2x-1)}{(x+4)(2x-1)} \div \frac{15}{(x-4)(x+4)}$

$= \frac{3(2x-1)}{(x+4)(2x-1)} \times \frac{(x-4)(x+4)}{15}$ ***[1 mark]***

$= \frac{3\cancel{(2x-1)}}{\cancel{(x+4)}\cancel{(2x-1)}} \times \frac{(x-4)\cancel{(x+4)}}{15}$

$= \frac{3(x-4)}{15} = \frac{x-4}{5}$ ***[1 mark]***

[4 marks available in total — as above]

19 Label CD as the height x, and AD as length y.
$x = 6\tan 34° = 4.0470...$ cm ***[1 mark]***
Use Pythagoras' theorem to find y:
$4.0470...^2 + y^2 = 5.4^2$, so $y^2 = 5.4^2 - 4.0470...^2 = 12.7813...$
$y = \sqrt{12.7813...} = 3.5751...$ cm ***[1 mark]***
Area of triangle $ABC = \frac{1}{2} \times (3.5751... + 6) \times 4.0470...$ ***[1 mark]***
$= 19.3754...$ cm² = 19.38 cm² (2 d.p.) ***[1 mark]***
[4 marks available in total — as above]

20 $(3n + 2)^2 - 1 = 9n^2 + 12n + 4 - 1 = 9n^2 + 12n + 3$ ***[1 mark]***
There is a common factor of 3 ***[1 mark]***, so rewrite this expression as $3(3n^2 + 4n + 1)$ which is a multiple of 3 for all positive integers. ***[1 mark]***
[3 marks available in total — as above]

21 If radius of shaded circle = r
Area of shaded circle = πr^2 ***[1 mark]***
Radius of sector = $2r$

Area of sector = $\frac{360° - a}{360°} \times \pi \times (2r)^2$

$= \frac{360° - a}{360°} \times 4\pi r^2$ ***[1 mark]***

Area of shaded circle = 30% of area of sector

$\pi r^2 = 0.3 \times \frac{360° - a}{360°} \times 4\pi r^2$ ***[1 mark]***

$1 = 1.2 \times \frac{360° - a}{360°}$

$\frac{360°}{1.2} = 360° - a$

$300° = 360° - a$

$a = 60°$ ***[1 mark]***

[4 marks available in total — as above]