

1. Find the exact solutions to the equations

(a) $\ln x + \ln 3 = \ln 6$,

(2)

(b) $e^x + 3e^{-x} = 4$.

(4)

7. The function f has domain $-2 \leq x \leq 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = f(x)$ is shown in Figure 1.

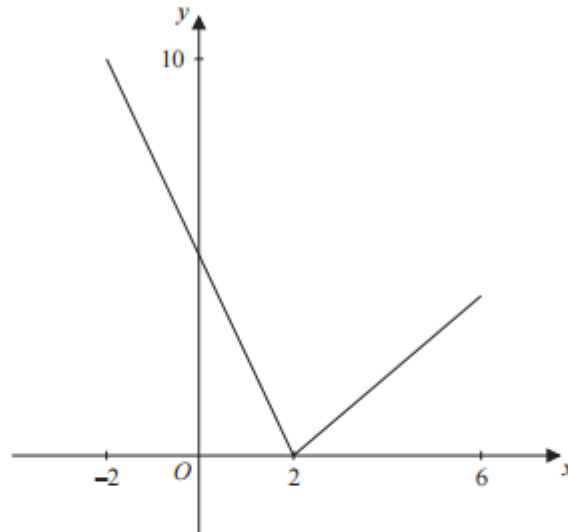


Figure 1

(a) Write down the range of f .

(1)

(b) Find $ff(0)$.

(2)

The function g is defined by

$$g : x \rightarrow \frac{4 + 3x}{5 - x}, \quad x \in \mathbb{R}, \quad x \neq 5$$

(c) Find $g^{-1}(x)$

(3)

(d) Solve the equation $gf(x) = 16$

(5)

6. The functions f and g are defined by

$$f : x \mapsto e^x + 2, \quad x \in \mathbb{R}$$

$$g : x \mapsto \ln x, \quad x > 0$$

- (a) State the range of f . (1)
- (b) Find $fg(x)$, giving your answer in its simplest form. (2)
- (c) Find the exact value of x for which $f(2x + 3) = 6$ (4)
- (d) Find f^{-1} , the inverse function of f , stating its domain. (3)
- (e) On the same axes sketch the curves with equation $y = f(x)$ and $y = f^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes. (4)
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4. A heated metal ball is dropped into a liquid. As the ball cools, its temperature, $T^\circ\text{C}$, t minutes after it enters the liquid, is given by

$$T = 400 e^{-0.05t} + 25, \quad t \geq 0.$$

- (a) Find the temperature of the ball as it enters the liquid. (1)
- (b) Find the value of t for which $T = 300$, giving your answer to 3 significant figures. (4)
- (c) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$. Give your answer in $^\circ\text{C}$ per minute to 3 significant figures. (3)
- (d) From the equation for temperature T in terms of t , given above, explain why the temperature of the ball can never fall to 20°C . (1)

2. Express

$$\frac{2x^2 + 3x}{(2x + 3)(x - 2)} - \frac{6}{x^2 - x - 2}$$

as a single fraction in its simplest form.

(7)

6. The function f is defined by

$$f: x \mapsto \frac{3-2x}{x-5}, \quad x \in \mathbb{R}, x \neq 5$$

(a) Find $f^{-1}(x)$.

(3)

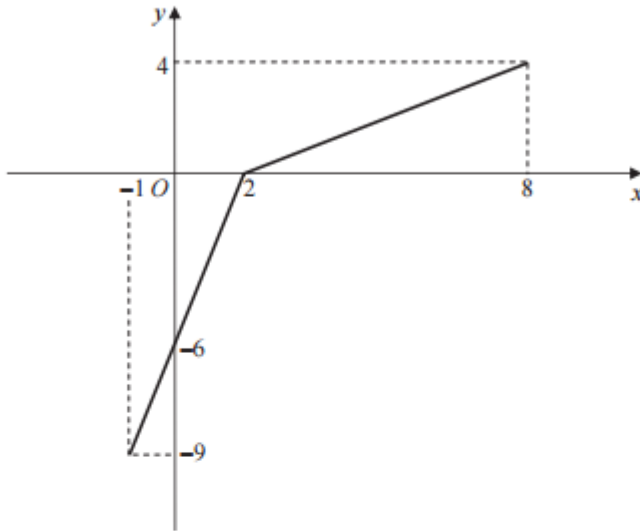


Figure 2

The function g has domain $-1 \leq x \leq 8$, and is linear from $(-1, -9)$ to $(2, 0)$ and from $(2, 0)$ to $(8, 4)$. Figure 2 shows a sketch of the graph of $y = g(x)$.

(b) Write down the range of g .

(1)

(c) Find $gg(2)$.

(2)

(d) Find $fg(8)$.

(2)

(e) On separate diagrams, sketch the graph with equation

(i) $y = |g(x)|$,

(ii) $y = g^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(f) State the domain of the inverse function g^{-1} .

(1)

5.

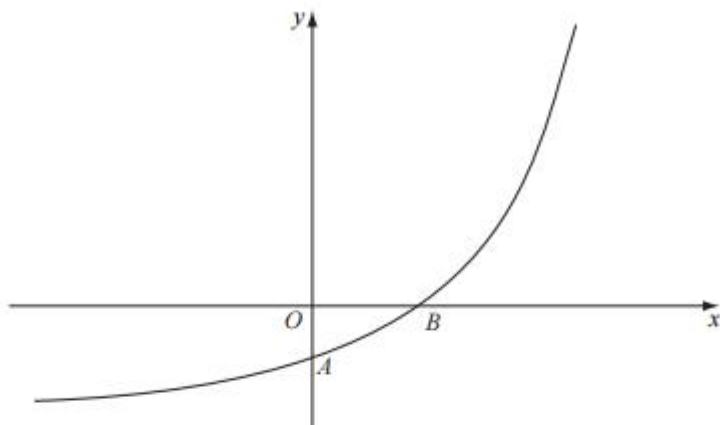


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$.

The curve meets the coordinate axes at the points $A(0, 1-k)$ and $B(\frac{1}{2} \ln k, 0)$, where k is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$, (3)

(b) $y = f^{-1}(x)$. (2)

Show on each sketch the coordinates, in terms of k , of each point at which the curve meets or cuts the axes.

Given that $f(x) = e^{2x} - k$,

(c) state the range of f , (1)

(d) find $f^{-1}(x)$, (3)

(e) write down the domain of f^{-1} . (1)