

C1 DIFFERENTIATION

Worksheet C

- 1 Find the gradient at the point with x -coordinate 3 on each of the following curves.
- a** $y = x^3$ **b** $y = 4x - x^2$ **c** $y = 2x^2 - 8x + 3$ **d** $y = \frac{3}{x} + 2$
- 2 Find the gradient of each curve at the given point.
- a** $y = 3x^2 + x - 5$ $(1, -1)$ **b** $y = x^4 + 2x^3$ $(-2, 0)$
c $y = x(2x - 3)$ $(2, 2)$ **d** $y = x^2 - 2x^{-1}$ $(2, 3)$
e $y = x^2 + 6x + 8$ $(-3, -1)$ **f** $y = 4x + x^{-2}$ $(\frac{1}{2}, 6)$
- 3 Evaluate $f'(4)$ when
- a** $f(x) = (x + 1)^2$ **b** $f(x) = x^{\frac{1}{2}}$ **c** $f(x) = x - 4x^{-2}$ **d** $f(x) = 5 - 6x^{\frac{3}{2}}$
- 4 The curve with equation $y = x^3 - 4x^2 + 3x$ crosses the x -axis at the points A , B and C .
- a** Find the coordinates of the points A , B and C .
b Find the gradient of the curve at each of the points A , B and C .
- 5 For the curve with equation $y = 2x^2 - 5x + 1$,
- a** find $\frac{dy}{dx}$,
b find the value of x for which $\frac{dy}{dx} = 7$.
- 6 Find the coordinates of the points on the curve with the equation $y = x^3 - 8x$ at which the gradient of the curve is 4.
- 7 A curve has the equation $y = x^3 + x^2 - 4x + 1$.
- a** Find the gradient of the curve at the point $P(-1, 5)$.
Given that the gradient at the point Q on the curve is the same as the gradient at the point P ,
b find, as exact fractions, the coordinates of the point Q .
- 8 Find an equation of the tangent to each curve at the given point.
- a** $y = x^2$ $(2, 4)$ **b** $y = x^2 + 3x + 4$ $(-1, 2)$
c $y = 2x^2 - 6x + 8$ $(1, 4)$ **d** $y = x^3 - 4x^2 + 2$ $(3, -7)$
- 9 Find an equation of the tangent to each curve at the given point. Give your answers in the form $ax + by + c = 0$, where a , b and c are integers.
- a** $y = 3 - x^2$ $(-3, -6)$ **b** $y = \frac{2}{x}$ $(2, 1)$
c $y = 2x^2 + 5x - 1$ $(\frac{1}{2}, 2)$ **d** $y = x - 3\sqrt{x}$ $(4, -2)$
- 10 Find an equation of the normal to each curve at the given point. Give your answers in the form $ax + by + c = 0$, where a , b and c are integers.
- a** $y = x^2 - 4$ $(1, -3)$ **b** $y = 3x^2 + 7x + 7$ $(-2, 5)$
c $y = x^3 - 8x + 4$ $(2, -4)$ **d** $y = x - \frac{6}{x}$ $(3, 1)$

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Worksheet C continued

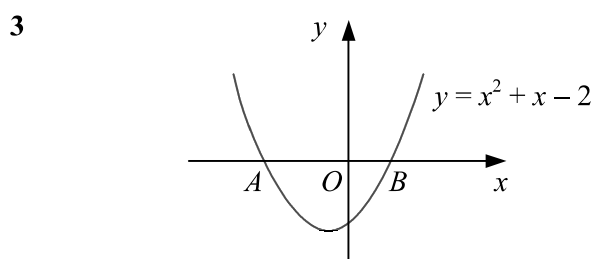
- 11 Find, in the form $y = mx + c$, an equation of
- the tangent to the curve $y = 3x^2 - 5x + 2$ at the point on the curve with x -coordinate 2,
 - the normal to the curve $y = x^3 + 5x^2 - 12$ at the point on the curve with x -coordinate -3 .
- 12 A curve has the equation $y = x^3 + 3x^2 - 16x + 2$.
- Find an equation of the tangent to the curve at the point $P(2, -10)$.
The tangent to the curve at the point Q is parallel to the tangent at the point P .
 - Find the coordinates of the point Q .
- 13 A curve has the equation $y = x^2 - 3x + 4$.
- Find an equation of the normal to the curve at the point $A(2, 2)$.
The normal to the curve at A intersects the curve again at the point B .
 - Find the coordinates of the point B .
- 14 $f(x) \equiv x^3 + 4x^2 - 18$.
- Find $f'(x)$.
 - Show that the tangent to the curve $y = f(x)$ at the point on the curve with x -coordinate -3 passes through the origin.
- 15 The curve C has the equation $y = 6 + x - x^2$.
- Find the coordinates of the point P , where C crosses the positive x -axis, and the point Q , where C crosses the y -axis.
 - Find an equation of the tangent to C at P .
 - Find the coordinates of the point where the tangent to C at P meets the tangent to C at Q .
- 16 The straight line l is a tangent to the curve $y = x^2 - 5x + 3$ at the point A on the curve. Given that l is parallel to the line $3x + y = 0$,
- find the coordinates of the point A ,
 - find the equation of the line l in the form $y = mx + c$.
- 17 The line with equation $y = 2x + k$ is a normal to the curve with equation $y = \frac{16}{x^2}$.
Find the value of the constant k .
- 18 A ball is thrown vertically downwards from the top of a cliff. The distance, s metres, of the ball from the top of the cliff after t seconds is given by $s = 3t + 5t^2$.
Find the rate at which the distance the ball has travelled is increasing when
- $t = 0.6$,
 - $s = 54$.
- 19 Water is poured into a vase such that the depth, h cm, of the water in the vase after t seconds is given by $h = kt^{\frac{1}{3}}$, where k is a constant. Given that when $t = 1$, the depth of the water in the vase is increasing at the rate of 3 cm per second,
- find the value of k ,
 - find the rate at which h is increasing when $t = 8$.

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Worksheet D

- 1 $f(x) = (x + 1)(x - 2)^2$.
- a Sketch the curve $y = f(x)$, showing the coordinates of any points where the curve meets the coordinate axes. (3)
- b Find $f'(x)$. (4)
- c Show that the tangent to the curve $y = f(x)$ at the point where $x = 1$ has the equation $y = 5 - 3x$. (3)

- 2 The curve C has the equation $y = x - 3x^{\frac{1}{2}} + 3$ and passes through the point $P(4, 1)$.
- a Show that the tangent to C at P passes through the origin. (5)
- The normal to C at P crosses the y -axis at the point Q .
- b Find the area of triangle OPQ , where O is the origin. (4)



The diagram shows the curve $y = x^2 + x - 2$. The curve crosses the x -axis at the points $A(a, 0)$ and $B(b, 0)$ where $a < b$.

- a Find the values of a and b . (3)
- b Show that the normal to the curve at A has the equation $x - 3y + 2 = 0$. (5)
- The tangent to the curve at B meets the normal to the curve at A at the point C .
- c Find the exact coordinates of C . (4)
- 4 Given that $y = \frac{x^2 - 6x - 3}{3x^{\frac{1}{2}}}$, show that $\frac{dy}{dx}$ can be expressed in the form $\frac{(x+a)^2}{bx^{\frac{3}{2}}}$, where a and b are integers to be found. (6)

- 5 The point A lies on the curve $y = \frac{12}{x^2}$ and the x -coordinate of A is 2.
- a Find an equation of the tangent to the curve at A . Give your answer in the form $ax + by + c = 0$, where a , b and c are integers. (5)
- b Verify that the point where the tangent at A intersects the curve again has the coordinates $(-1, 12)$. (3)
- 6 A curve has the equation $y = 2 + 3x + kx^2 - x^3$ where k is a constant.
- Given that the gradient of the curve is -6 at the point P where $x = -1$,
- a find the value of k . (4)
- Given also that the tangent to the curve at the point Q is parallel to the tangent at P ,
- b find the length PQ , giving your answer in the form $k\sqrt{5}$. (5)

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Worksheet D continued

7 Differentiate $x^2 + \frac{1}{2x}$ with respect to x . (3)

8 A curve has the equation $y = 2x^2 - 7x + 1$ and the point A on the curve has x -coordinate 2.

a Find an equation of the tangent to the curve at A . (4)

The normal to the curve at the point B is parallel to the tangent at A .

b Find the coordinates of B . (3)

9 $y = x^2 + 3x^{\frac{1}{2}}$.

a Find $\frac{dy}{dx}$. (2)

b Show that $2x \frac{d^2y}{dx^2} + \frac{dy}{dx} - 6x = 0$. (4)

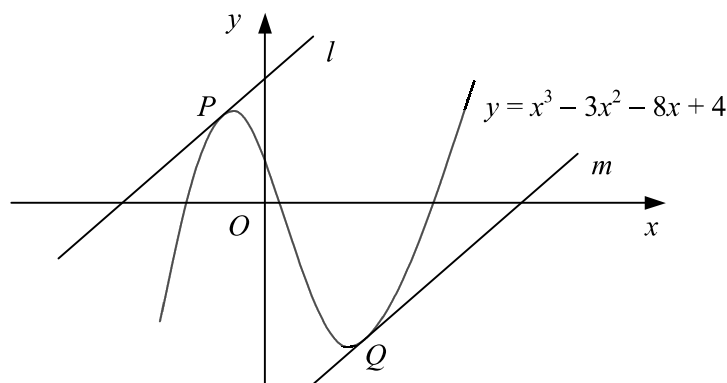
10 A curve has the equation $y = 2 + \frac{4}{x}$.

a Find an equation of the normal to the curve at the point $M(4, 3)$. (5)

The normal to the curve at M intersects the curve again at the point N .

b Find the coordinates of the point N . (5)

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The diagram shows the curve with equation $y = x^3 - 3x^2 - 8x + 4$.

The straight line l is the tangent to the curve at the point $P(-1, 8)$.

a Find an equation of line l . (4)

The straight line m is parallel to l and is the tangent to the curve at the point Q .

b Find an equation of line m . (4)

c Find an equation of the normal to the curve at P . (2)

d Hence, or otherwise, show that the distance between lines l and m is $16\sqrt{2}$. (4)

12 A curve has the equation $y = \sqrt{x}(k-x)$, where k is a constant.

Given that the gradient of the curve is $\sqrt{2}$ at the point P where $x = 2$,

a find the value of k , (5)

b show that the normal to the curve at P has the equation

$$x + \sqrt{2}y = c,$$

where c is an integer to be found. (5)