Write your name here

Surname	Other names

Pearson
Edexcel GCE

Centre Number

Candidate Number

A level Mathematics

Practice Paper
Pure Mathematics - Trigonometry (part 1)

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 70.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. In the triangle ABC, $AB = 11$ cm, $BC = 7$ cm and $CA = 8$ cm.

(*a*) Find the size of angle C, giving your answer in radians to 3 significant figures.

(3)

(*b*) Find the area of triangle ABC, giving your answer in cm^2 to 3 significant figures.

(3)

(Total 6 marks)

2.

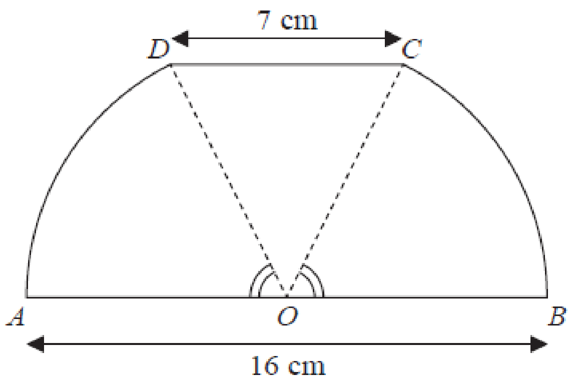

Figure 1

Figure 1 shows a sketch of a design for a scraper blade. The blade *AOBCDA* consists of an isosceles triangle *COD* joined along its equal sides to sectors *OBC* and *ODA* of a circle with centre *O* and radius 8 cm. Angles *AOD* and *BOC* are equal. *AOB* is a straight line and is parallel to the line *DC*. *DC* has length 7 cm.

(*a*) Show that the angle *COD* is 0.906 radians, correct to 3 significant figures.

(2)

(*b*) Find the perimeter of *AOBCDA*, giving your answer to 3 significant figures.

(3)

(*c*) Find the area of *AOBCDA*, giving your answer to 3 significant figures.

(3)

(Total 8 marks)

3.

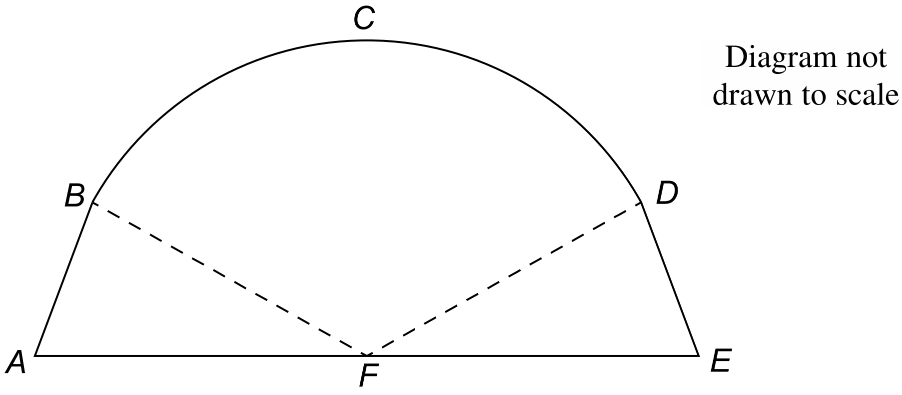

Figure 2

Figure 2 is a sketch representing the cross-section of a large tent *ABCDEF*.

AB and *DE* are line segments of equal length.

Angle *FAB* and angle *DEF* are equal.

F is the midpoint of the straight line *AE* and *FC* is perpendicular to *AE*.

BCD is an arc of a circle of radius 3.5 m with centre at *F*.

It is given that

$$AF = FE = 3.7 \text{ m}$$

$$BF = FD = 3.5 \text{ m}$$

$$\text{angle } BFD = 1.77 \text{ radians}$$

Find

(*a*) the length of the arc *BCD* in metres to 2 decimal places,

(2)

(*b*) the area of the sector *FBCD* in m^2 to 2 decimal places,

(2)

(*c*) the total area of the cross-section of the tent in m^2 to 2 decimal places.

(4)

(Total 8 marks)

Turn over

4.

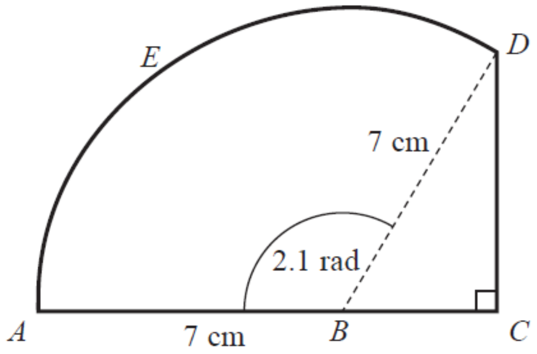

Figure 3

Figure 3 shows the shape *ABCDEA* which consists of a right-angled triangle *BCD* joined to a sector *ABDEA* of a circle with radius 7 cm and centre *B*.

A, *B* and *C* lie on a straight line with *AB* = 7 cm.

Given that the size of angle *ABD* is exactly 2.1 radians,

(*a*) find, in cm, the length of the arc *DEA*,

(2)

(*b*) find, in cm, the perimeter of the shape *ABCDEA*, giving your answer to 1 decimal place.

(4)

(Total 6 marks)

Turn over

5.

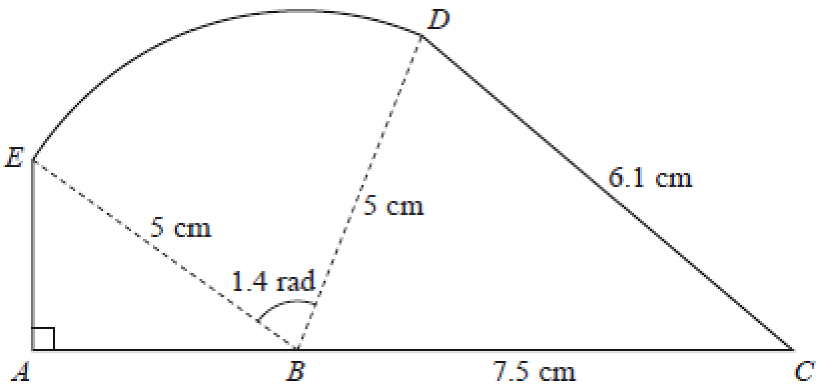

Figure 4

The shape *ABCDEA*, as shown in Figure 4, consists of a right-angled triangle *EAB* and a triangle *DBC* joined to a sector *BDE* of a circle with radius 5 cm and centre *B*.

The points *A*, *B* and *C* lie on a straight line with *BC* = 7.5 cm.

Angle *EAB* = $\frac{\pi}{2}$ radians, angle *EBD* = 1.4 radians and *CD* = 6.1 cm.

(*a*) Find, in cm^2, the area of the sector *BDE*.

(2)

(*b*) Find the size of the angle *DBC*, giving your answer in radians to 3 decimal places.

(2)

(*c*) Find, in cm^2, the area of the shape *ABCDEA*, giving your answer to 3 significant figures.

(5)

(Total 9 marks)

6.

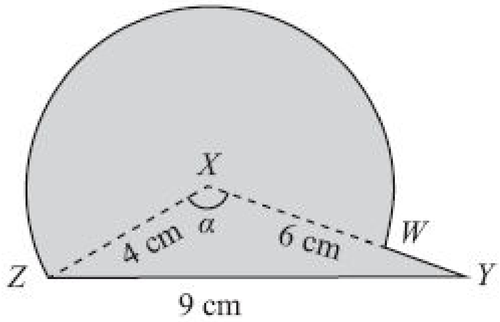

Figure 5

The triangle XYZ in Figure 1 has $XY = 6$ cm, $YZ = 9$ cm, $ZX = 4$ cm and angle $ZXY = \alpha$.
The point W lies on the line XY.

The circular arc ZW, in Figure 5 is a major arc of the circle with centre X and radius 4 cm.

(*a*) Show that, to 3 significant figures, $\alpha = 2.22$ radians.

(2)

(*b*) Find the area, in cm^2, of the major sector $XZWX$.

(3)

The region enclosed by the major arc ZW of the circle and the lines WY and YZ is shown shaded in Figure 5.

Calculate

(*c*) the area of this shaded region,

(3)

(*d*) the perimeter $ZWYZ$ of this shaded region.

(4)

(Total 12 marks)

Turn over

7.

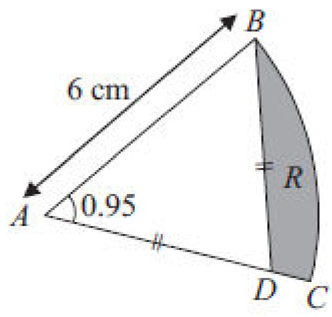

Figure 6

Figure 6 shows *ABC*, a sector of a circle of radius 6 cm with centre *A*. Given that the size of angle *BAC* is 0.95 radians, find

(*a*) the length of the arc *BC*,

(2)

(*b*) the area of the sector *ABC*.

(2)

The point *D* lies on the line *AC* and is such that *AD* = *BD*. The region *R*, shown shaded in Figure 6, is bounded by the lines *CD*, *DB* and the arc *BC*.

(*c*) Show that the length of *AD* is 5.16 cm to 3 significant figures.

(2)

Find

(*d*) the perimeter of *R*,

(2)

(*e*) the area of *R*, giving your answer to 2 significant figures.

(4)

(Total 12 marks)

8.

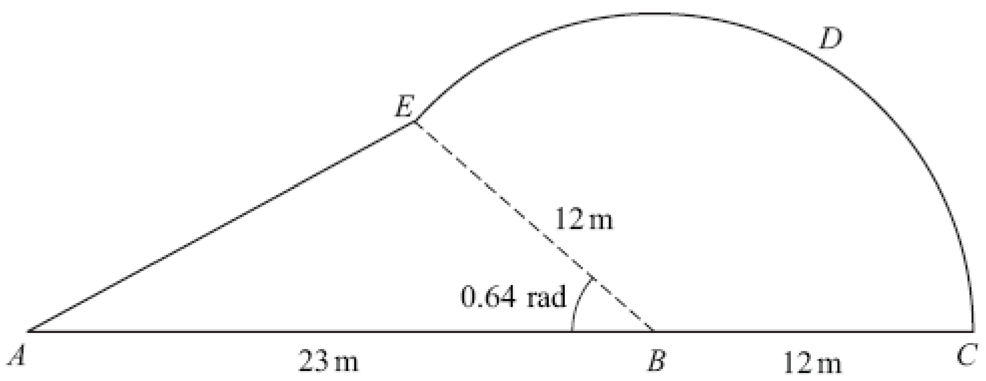

Figure 7

Figure 7 shows a plan view of a garden.

The plan of the garden *ABCDEA* consists of a triangle *ABE* joined to a sector *BCDE* of a circle with radius 12 m and centre *B*.

The points *A*, *B* and *C* lie on a straight line with *AB* = 23 m and *BC* = 12 m.

Given that the size of angle *ABE* is exactly 0.64 radians, find

(*a*) the area of the garden, giving your answer in m^2, to 1 decimal place,

(4)

(*b*) the perimeter of the garden, giving your answer in metres, to 1 decimal place.

(5)

(Total 9 marks)

TOTAL FOR PAPER: 70 MARKS

Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

A level Mathematics

Practice Paper
Pure Mathematics - Trigonometry (part 2)

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. (*a*) Express $2 \cos \theta - \sin \theta$ in the form $R \cos (\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$ Give the exact value of R and give the value of α to 2 decimal places.

(3)

(*b*) Hence solve, for $0 \leq \theta < 360°$,

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15.$$

Give your answers to one decimal place.

(5)

(*c*) Use your solutions to parts (*a*) and (*b*) to deduce the smallest positive value of θ for which

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15.$$

Give your answer to one decimal place.

(2)

(Total 10 marks)

---

2. $$g(\theta) = 4 \cos 2\theta + 2 \sin 2\theta.$$

Given that $g(\theta) = R \cos (2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(*a*) find the exact value of R and the value of α to 2 decimal places.

(3)

(*b*) Hence solve, for $-90° < \theta < 90°$,

$$4 \cos 2\theta + 2 \sin 2\theta = 1,$$

giving your answers to one decimal place.

(5)

Given that k is a constant and the equation $g(\theta) = k$ has no solutions,

(*c*) state the range of possible values of k.

(2)

(Total 10 marks)

---

3. Given that

$$2\cos(x+50)° = \sin(x+40)°.$$

(*a*) Show, without using a calculator, that

$$\tan x° = \frac{1}{3}\tan 40°.$$

(4)

(*b*) Hence solve, for $0 \leq \theta < 360$,

$$2\cos(2\theta+50)° = \sin(2\theta+40)°,$$

giving your answers to 1 decimal place.

(4)

(Total 8 marks)

---

4. Find all the solutions of

$$2\cos 2\theta = 1 - 2\sin\theta$$

in the interval $0 \leq \theta < 360°$.

(Total 6 marks)

Turn over

5. (*a*) Write $5\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants,

$R > 0$ and $0 \leqslant \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

(*b*) Show that the equation

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

can be rewritten in the form

$$5\cos 2x - 2\sin 2x = c$$

where c is a positive constant to be determined.

(2)

(*c*) Hence or otherwise, solve, for $0 \leqslant x < \pi$,

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

giving your answers to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(Total 9 marks)

---

6. (*a*) Express $6\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(4)

(*b*)

$$\mathrm{p}(\theta) = \frac{4}{12 + 6\cos\theta + 8\sin\theta}, \quad 0 \leq \theta \leq 2\pi.$$

Calculate

(i) the maximum value of $\mathrm{p}(\theta)$,

(ii) the value of θ at which the maximum occurs.

(4)

(Total 8 marks)

---

7. (i) Without using a calculator, find the exact value of

$$(\sin 22.5° + \cos 22.5°)^2.$$

You must show each stage of your working.

(5)

(ii) (*a*) Show that $\cos 2\theta + \sin \theta = 1$ may be written in the form

$$k \sin^2 \theta - \sin \theta = 0, \text{ stating the value of } k.$$

(2)

(*b*) Hence solve, for $0 \leq \theta < 360°$, the equation

$$\cos 2\theta + \sin \theta = 1.$$

(4)

(Total 11 marks)

8.

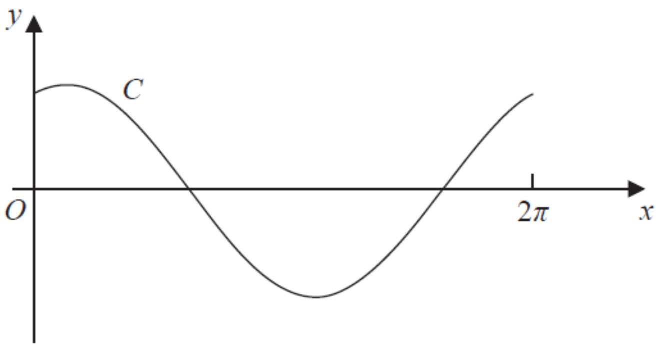

Figure 1

Figure 1 shows the curve C, with equation $y = 6 \cos x + 2.5 \sin x$ for $0 \leq x \leq 2\pi$.

(*a*) Express $6 \cos x + 2.5 \sin x$ in the form $R \cos(x - \alpha)$, where R and α are constants with $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your value of α to 3 decimal places.

(3)

(*b*) Find the coordinates of the points on the graph where the curve C crosses the coordinate axes.

(3)

A student records the number of hours of daylight each Sunday throughout the year. She starts on the last Sunday in May with a recording of 18 hours, and continues until her final recording 52 weeks later.

She models her results with the continuous function given by

$$H = 12 + 6\cos\left(\frac{2\pi t}{52}\right) + 2.5\sin\left(\frac{2\pi t}{52}\right), \qquad 0 \leq t \leq 52$$

where H is the number of hours of daylight and t is the number of weeks since her first recording.

Use this function to find

(*c*) the maximum and minimum values of H predicted by the model,

(3)

(*d*) the values for t when $H = 16$, giving your answers to the nearest whole number.

[*You must show your working. Answers based entirely on graphical or numerical methods are not acceptable.*]

(6)

(Total 15 marks)

9.

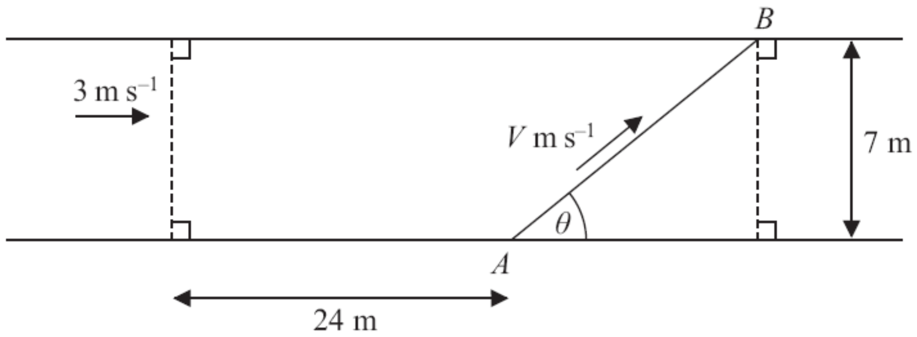

Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1}.

Kate is 24 m ahead of John when she starts to cross the road from the fixed point A.
John passes her as she reaches the other side of the road at a variable point B, as shown in Figure 2.

Kate's speed is V m s^{-1} and she moves in a straight line, which makes an angle θ, $0 < \theta < 150°$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24\sin\theta + 7\cos\theta}, \qquad 0 < \theta < 150°$$

(a) Express $24\sin\theta + 7\cos\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(3)

Given that θ varies,

(b) find the minimum value of V.

(2)

Given that Kate's speed has the value found in part (b),

(c) find the distance AB.

(3)

Given instead that Kate's speed is 1.68 m s^{-1},

(d) find the two possible values of the angle θ, given that $0 < \theta < 150°$.

(6)

(Total 14 marks)

---

10. (*a*) Express $2 \sin \theta - 4 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(3)

$$H(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$$

Find

(*b*) (i) the maximum value of $H(\theta)$,

(ii) the smallest value of θ, for $0 \leq \theta \leq \pi$, at which this maximum value occurs.

(3)

Find

(*c*) (i) the minimum value of $H(\theta)$,

(ii) the largest value of θ, for $0 \leq \theta \leq \pi$, at which this minimum value occurs.

(3)

(Total 9 marks)

TOTAL FOR PAPER: 100 MARKS

Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

A level Mathematics

Practice Paper
Pure Mathematics - Trigonometry (part 3)

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
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- Fill in the boxes at the top of this page with your name, centre number and candidate number.
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- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 96.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. (i) (*a*) Show that $2 \tan x - \cot x = 5 \operatorname{cosec} x$ may be written in the form

$$a \cos^2 x + b \cos x + c = 0$$

stating the values of the constants a, b and c.

(4)

(*b*) Hence solve, for $0 \leq x < 2\pi$, the equation

$$2 \tan x - \cot x = 5 \operatorname{cosec} x$$

giving your answers to 3 significant figures.

(4)

(ii) Show that

$$\tan \theta + \cot \theta \equiv \lambda \operatorname{cosec} 2\theta, \quad \theta = \frac{n\pi}{2}, \quad n \in \mathbb{Z}$$

stating the value of the constant λ.

(4)

(Total 12 marks)

---

2. (*a*) Express $4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta$ in terms of $\sin \theta$ and $\cos \theta$.

(2)

(*b*) Hence show that

$$4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta = \sec^2 \theta .$$

(4)

(*c*) Hence or otherwise solve, for $0 < \theta < \pi$,

$$4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta = 4$$

giving your answers in terms of π.

(3)

(Total 9 marks)

---

3. (*a*) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan\,\theta, \qquad \theta \neq 90n°, \quad n \in \mathbb{Z}.$$

(4)

(*b*) Hence, or otherwise,

(i) show that $\tan 15° = 2 - \sqrt{3}$,

(3)

(ii) solve, for $0 < x < 360°$,

$$\text{cosec}\, 4x - \cot 4x = 1.$$

(5)

(Total 12 marks)

---

4. (*a*) Show that

$$\text{cosec}\, 2x + \cot 2x = \cot x, \qquad x \neq 90n°, \qquad n \in \mathbb{Z}$$

(5)

(*b*) Hence, or otherwise, solve, for $0 \leq \theta < 180°$,

$$\text{cosec}\,(4\theta + 10°) + \cot\,(4\theta + 10°) = \sqrt{3}$$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

(Total 10 marks)

---

5. (*a*) Prove that

$$2\cot 2x + \tan x \equiv \cot x, \qquad x \neq \frac{n\pi}{2}, \qquad n \in \mathbb{Z}$$

(4)

(*b*) Hence, or otherwise, solve, for $-\pi \leq x < \pi$,

$$6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2.$$

Give your answers to 3 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(Total 10 marks)

---

6. (*a*) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, \qquad A \neq \frac{(2n+1)\pi}{4}, \qquad n \in \mathbb{Z}.$$

(5)

(*b*) Hence solve, for $0 \leq \theta < 2\pi$,

$$\sec 2\theta + \tan 2\theta = \frac{1}{2}.$$

Give your answers to 3 decimal places.

(4)

(Total 9 marks)

---

7. $$f(x) = 7 \cos 2x - 24 \sin 2x.$$

Given that $f(x) = R \cos (2x + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(*a*) find the value of R and the value of α.

(3)

(*b*) Hence solve the equation

$$7 \cos 2x - 24 \sin 2x = 12.5$$

for $0 \leq x < 180°$, giving your answers to 1 decimal place.

(5)

(*c*) Express $14 \cos^2 x - 48 \sin x \cos x$ in the form $a \cos 2x + b \sin 2x + c$, where a, b, and c are constants to be found.

(2)

(*d*) Hence, using your answers to parts (*a*) and (*c*), deduce the maximum value of

$$14 \cos^2 x - 48 \sin x \cos x.$$

(2)

(Total 12 marks)

---

8. (*a*) Starting from the formulae for $\sin (A + B)$ and $\cos (A + B)$, prove that

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

(4)

(*b*) Deduce that

$$\tan \left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3} \tan \theta}{\sqrt{3} - \tan \theta}.$$

(3)

(*c*) Hence, or otherwise, solve, for $0 \leq \theta \leq \pi$,

$$1 + \sqrt{3} \tan \theta = (\sqrt{3} - \tan \theta) \tan (\pi - \theta).$$

Give your answers as multiples of π.

(6)

(Total 13 marks)

---

9. (*a*) Prove that

$$\sin 2x - \tan x \equiv \tan x \cos 2x, \qquad x \neq (2n+1)90°, \quad n \in \mathbb{Z}$$

(4)

(*b*) Given that $x \neq 90°$ and $x \neq 270°$, solve, for $0 \leqslant x < 360°$,

$$\sin 2x - \tan x = 3 \tan x \sin x$$

Give your answers in degrees to one decimal place where appropriate.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(5)

(Total 9 marks)

TOTAL FOR PAPER: 96 MARKS