Changing the Subject: Advanced

Video 8 on www.corbettmaths.com

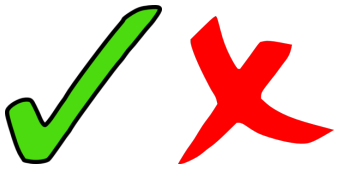
Question 3: Make c the subject of the following

(a) $w = \frac{ac}{a - c}$ (b) $w = 6 + \frac{a}{c + 2}$

Apply

Question 1: The cosine rule is $a^2 = b^2 + c^2 - 2bc \cos A$.
Make cos A the subject.

Answers



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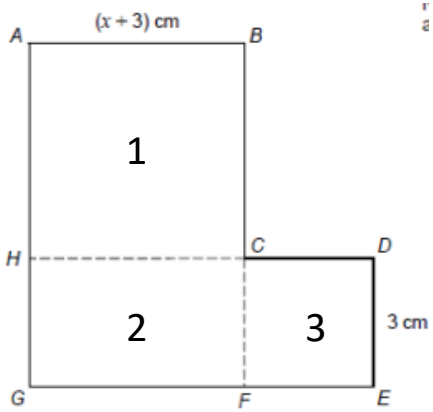
Forming and Solving Equations

Hard example

ABCH is a square. HCFG is a rectangle. CDEF is a square.



Show that the total area of the L-shape in cm² is $x^2 + 9x + 27$

Remember when you multiply two expressions like this together it forms double brackets

Area of square 1: $(x + 3) \times (x + 3)$
$= (x + 3)(x + 3)$
$= x^2 + 6x + 9$

Area of rectangle 2: $3 \times (x + 3)$
$= 3x + 9$

Area of square 3: $3 \times 3 = 9$

Total area $= x^2 + 6x + 9 + 3x + 9 + 9$
$= x^2 + 9x + 27$

x+2

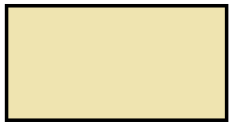
The area of this shape is 36cm².
Find the value of x.

(x+7)

(x - 2)

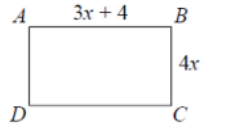
The area of this shape is 22cm².
Find the value of x.

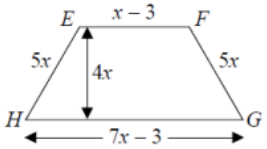
(2x - 9)



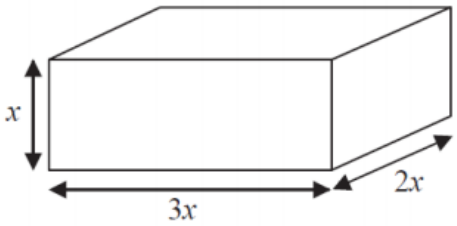
(x - 2)

The area of this shape is 42cm².
Form an equation for the area of this shape
Solve this equation and obtain a suitable value for x

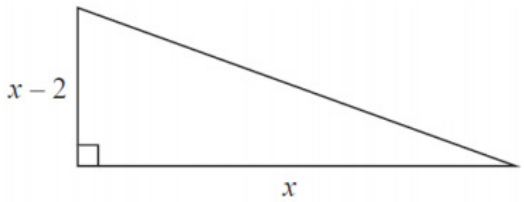




ABCD is a rectangle. EFGH is a trapezium.
All measurements are in centimetres.
The perimeters of these two shapes are the same.
Work out the area of the rectange.



All measurements are in centimetres.
X is an integer.
The total volume of the cuboid is less that 900cm³
Show that x ≤ 5



The area of the triangle is 2.5 cm².
Find the perimeter of the triangle. Give your answer correct to 3 significant figures.

Mixed exam questions

Q1. June 2015 unit 1

Here is a shape.

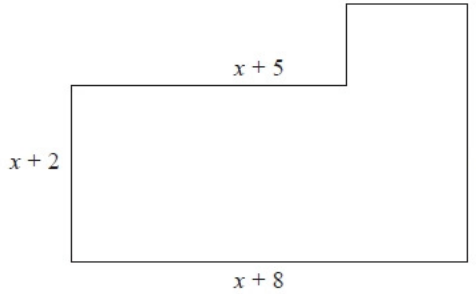


Diagram **NOT** accurately drawn

All the measurements are in centimetres. All the corners are right angles.
The area of the shape is *A* cm².
Find a formula for *A* in terms of *x*.
Give your answer in its simplest form.

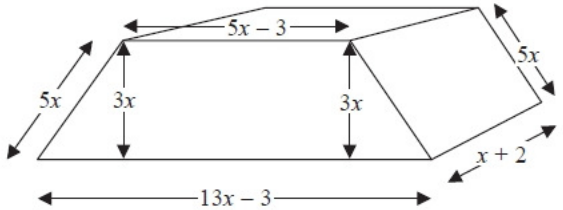
A = ...

(Total for question = 4 marks)

Q2. June 2014 unit 1

***** This shape is a solid prism. The cross section of the prism is a trapezium.

Diagram **NOT** accurately drawn



Show that the total surface area of the prism is $82x^2 + 32x - 12$

(Total for Question is 4 marks)

Q3. **November 2014 paper 1**

The diagram shows the plan of a floor.

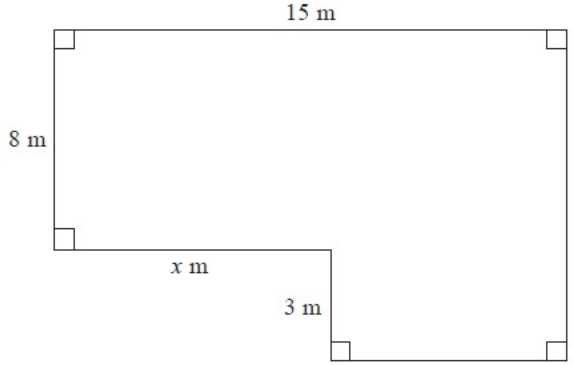


Diagram **NOT** accurately drawn

The area of the floor is 138 m^2.
Work out the value of *x*.

.....

(Total for Question is 4 marks)

Examiner's Report Key notes

- Be careful when subtracting expressions. E.g. (x + 8) − (x + 5) is actually x + 8 – x – 5.
- Remember to set out your working out carefully. Make it clear to the examiner what you are trying to calculate. On this type of question it may halp to number each section of the shape.

Answers

$(x + 2)(x + 2) = 36$ $x^2 + 4x + 4 = 36$ $x^2 + 4x - 32 = 0$ $(x + 8)(x – 4) = 0$ $X = -8$ or 4 so $x = 4$	$(x + 7)(x - 2) = 22$ $x^2 + 5x - 14 = 22$ $x^2 + 5x - 36 = 0$ $(x + 9)(x – 4) = 0$ $X = -9$ or 4 so $x = 4$
$(2x – 9)(x – 2) = 42$ $2x^2 - 13x + 18 = 42$ $2x^2 - 13x - 24 = 0$ $(2x + 3)(x – 8) = 0$ so $x = 8$	
Perimeter of the rectangle: $3x + 4 + 3x + 4 + 4x + 4x = 14x + 8$ Perimeter of the trapezium: $7x – 3 + 5x + 5x + x – 3 = 18x – 6$ The perimeters are equal, so $14x + 8 = 18x – 6$ $8 = 4x – 6$ $14 = 4x$ so $x = 3.5$ Area of the rectangle = $(3x + 4) \times 4x$ $= (10.5 + 4) \times 14 = 14.5 \times 14 = 187cm^2$	
Volume = $x \times 3x \times 2x = 6x^3$ $6x^3 < 900$ $x^3 < 150$ the nearest cube number is 120, which is 5 cubed so $x \leq 5$	
$x^2 - 2x = 2.5$ $x^2 - 2x - 2.5 = 0$ Using the quadratic formula, $x = 2.87$ To find the perimeter, we need to use Pythagoras first. $X – 2 = 2.87 – 2 = 0.87$ $c^2 = 0.87^2 + 2.87^2$ $c^2 = 8.9938$ $c = 3.00$ Perimeter = $0.87 + 2.87 + 3 = 6.74cm$	

Exam questions

Q1.

Answer	Mark	Notes
$x^2+10x+22$	4	M1 for $(x+8) – (x+5)$ (=3) or $(x+4) – (x+2)$ (=2) M1 for area of one rectangle eg $(x+2)(x+5)$ $(= x^2+7x+10)$ M1 for complete method to find area e.g. $(x+2)(x+5) + 3(x+4)$ $(= x^2+7x+10 + 3x+12)$ A1 cao

Q2.

Working	Answer	Mark	Notes
Front or Back: $\frac{1}{2} \times 3x(13x – 3 + 5x – 3)$ $= 27x^2 – 9x$ or $\frac{1}{2}(4x)(3x) + 3x(5x – 3)$ **Top**: $(5x – 3)(x + 2)$ $= 5x^2 + 7x – 6$ **Bottom**: $(13x–3)(x + 2)$ $= 13x^2 + 23x – 6$ **Each Side**: $5x(x + 2)$ $= 5x^2 + 10x$ **Total SA** = $2(27x^2 – 9x) +$ $2(5x^2 + 10x) + (5x^2 + 7x – 6)$ $+ (13x^2 + 23x – 6)$ $= (54 + 10 + 5 + 13)\,x^2$ $+ (–18 + 20 + 7 + 23)\,x$ $+ (– 6 – 6)$	$82x^2 + 32x – 12$	4	M1 finds the area of at least 2 faces (condone omission of brackets) M1 writes a correct algebraic expression for the area of at least 3 different faces M1 correct expressions for all 6 faces and adds C1 (dep on M3) for correct algebraic expression as a correct summary

Q3.

Answer	Mark	Notes
9	4	M1 for method to find area of one rectangle, eg 15×8 (=120) or 15×11 (=165) M1 (dep) for subtracting from/by given area, eg (138 – "120") (=18) or "165" – 138 (=27) M1 for final step from complete method shown, eg 15 – "18" ÷ 3 or "27" ÷ 3 A1 cao **OR** M1 for a correct expression for the area of one rectangle, eg $(8 + 3) \times (15 – x)$ or $8 \times x$ M1 (dep) for a correct equation eg $(8 + 3) \times (15 – x) + 8 \times x = 138$ M1 for correct method to isolate x, eg $3x = 27$ A1 cao

1. Write $\sqrt{48}$ in the form $k\sqrt{3}$, where k is an integer.

(2 marks)

2. Write $\sqrt{50}$ in the form $k\sqrt{2}$, where k is an integer.

(2 marks)

3. Write $5\sqrt{27}$ in the form $k\sqrt{3}$, where k is an integer.

(2 marks)

4. Write $7\sqrt{20}$ in the form $k\sqrt{5}$, where k is an integer.

(2 marks)

5. Expand and simplify $(2+\sqrt{3})(2-\sqrt{3})$

(2 marks)

6. Write $(3+\sqrt{5})^2$ in the form $a+b\sqrt{5}$ where a and b are integers

(2 marks)

7. Expand and simplify $(2+\sqrt{5})(1-\sqrt{5})$

(2 marks)

8. Write $(3-\sqrt{2})^2$ in the form $a+b\sqrt{2}$ where a and b are integers

(2 marks)

9. Expand and simplify $(2+\sqrt{3})^2-(2-\sqrt{3})^2$

(2 marks)

10. Rationalise the denominator $\frac{6}{\sqrt{3}}$

(2 marks)

11. Rationalise the denominator $\frac{x}{\sqrt{x}}$

(2 marks)

12. Rationalise the denominator $\frac{1+\sqrt{5}}{\sqrt{2}}$

(2 marks)

13. Simplify $\frac{3+\sqrt{6}}{3}$

(2 marks)

14. Simplify fully $\frac{(4+2\sqrt{3})(4-2\sqrt{3})}{\sqrt{11}}$ You must show all your working.

(3 marks)

15. Show that $\frac{5+2\sqrt{3}}{2+\sqrt{3}}$ can be written as $4-\sqrt{3}$

(3 marks)

16. Show that $\frac{3\sqrt{3}+3}{3+\sqrt{3}}$ can be written as $\sqrt{3}$

(3 marks)

17. Show that $\frac{1}{\frac{1}{\sqrt{2}}+\sqrt{2}}$ can be written as $\frac{\sqrt{2}}{3}$

(3 marks)

18. Simplify fully $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$

(2 marks)

19. Simplify fully $(2a+\sqrt{b})^2$

(2 marks)

Corbett maths

Functions

Videos 369, 370 on Corbettmaths

Question 16: Given $g(x) = \dfrac{3x+1}{2}$

(a) Find $g^{-1}(x)$

(b) Calculate the value of $g^{-1}(11)$

Question 17: Given $f(x) = \dfrac{4x}{9} - 8$

(a) Find $f^{-1}(x)$

(b) Calculate the value of $f^{-1}(-10)$

Apply

Question 1: Given $f(x) = 5x + 7$ and $g(x) = 3x - 18$

Find the value of a such that $f(a) = g(a)$

Question 2: Given $f(x) = x^2 + 9$ and $g(x) = x + 21$

Find the values of a such that $f(a) = g(a)$

Question 3: Given $f(x) = \dfrac{x+1}{3}$ and $g(x) = \dfrac{2}{x+2}$

Find the values of a such that $f(a) = g(a)$

Question 4: Given $f(x) = x^2 + 4x - 1$

Express the following in the form $ax^2 + bx + c$

(a) $f(x+2)$ (b) $f(x-1)$ (c) $f(2x)$

(d) $f(3x)$ (e) $f(2x-1)$ (f) $f(4x+3)$

Corbett maths

Functions

Videos 369, 370 on Corbettmaths

Question 5: The function f is such that $f(x) = kx + 7$

The function g is such that $g(x) = 3x - 2$

Given that $gf(1) = 34$

Work out the value of k

Question 6: The function g is such that $f(x) = \dfrac{kx+2}{4}$

The function h is such that $g(x) = 2x + 5$

Given that $fg(4) = -9.25$

Work out the value of k

Question 7: For all values of x

$$f(x) = x^2 + 5$$

$$g(x) = x - 4$$

Solve $fg(x) = gf(x)$

Question 8: $f(x) = x^2 + 3x + 8$

Show that $f(x+1) - f(x) = 2x + 4$

Answers

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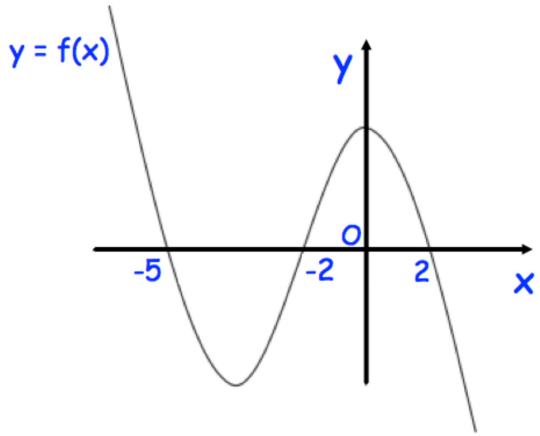
8.



The graph of y = f(x) cuts the x axis when x = –5, –2 and 2

Write down the coordinates of the points where these graphs cut the x axis.

(a) y = f(–x)

.....

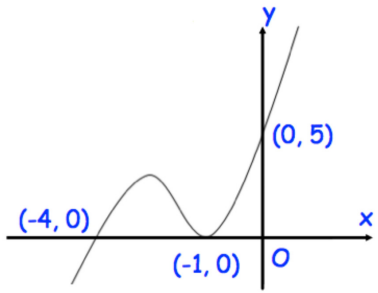
(2)

(b) y = f(x + 2)

.....

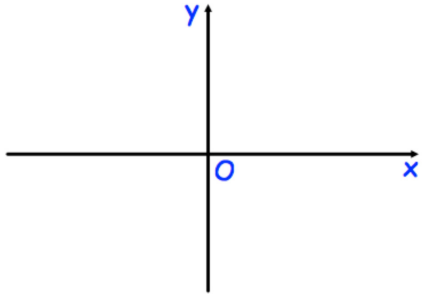
(2)

9. Shown below is the curve with equation y = f(x).
The curve passes through the points (–4, 0), (–1, 0) and (0, 5)



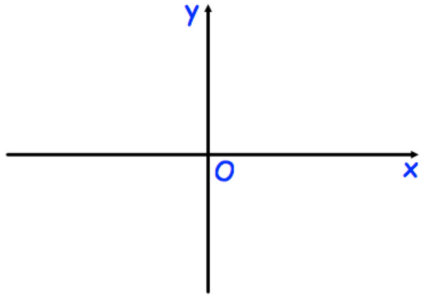
Sketch the curve with equation:

(a) y = f(x – 1)



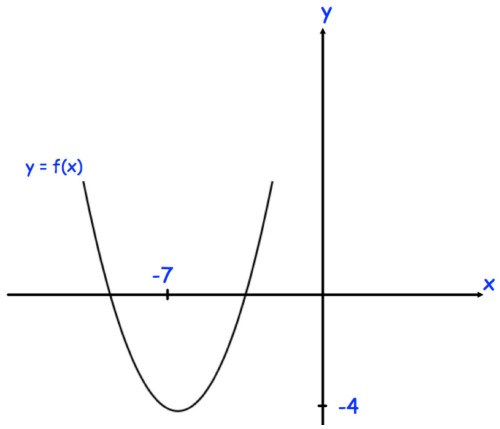
(2)

(b) y = –f(x)



(2)

10. Shown below is a sketch of a curve with equation y = f(x).
The curve has a minimum point at (-7, -4).



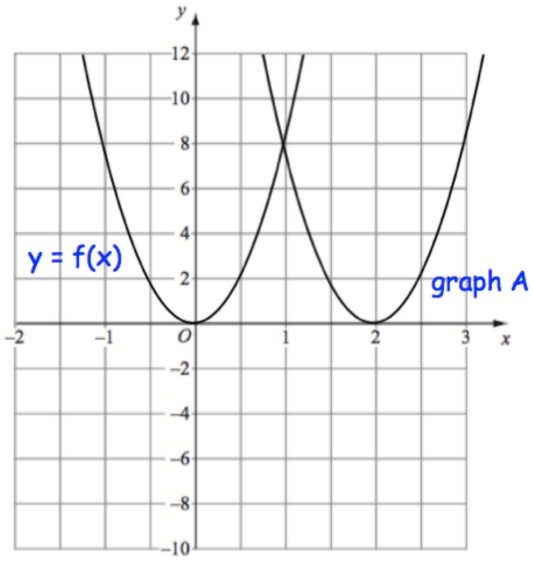
The graph of y = f(x) + a has a minimum point at (–7, 0), where a is a constant.

Write down the value of a.

.....

(1)

11. The graph of y = f(x) is shown on the grid.



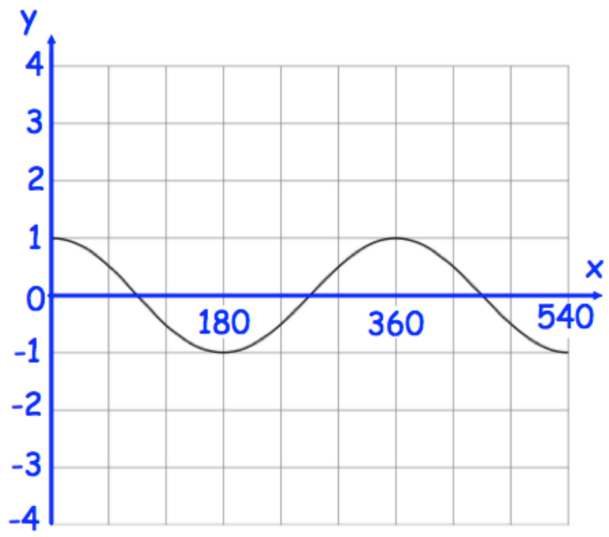
The graph A is a translation of the graph y = f(x)

Write down the equation of graph A.

.....

(2)

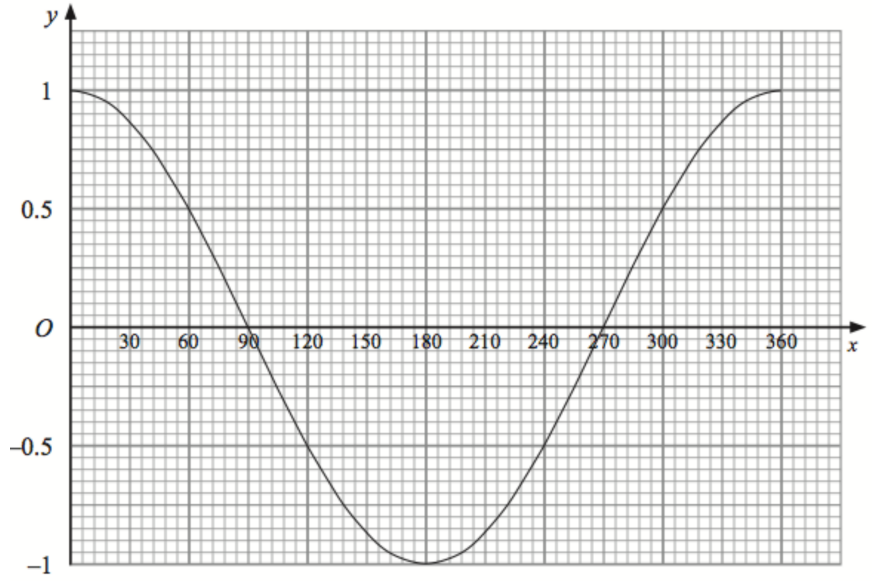
12. Shown below is the graph of y = cos x



On the grid, sketch the graph of y = 3 + cos x for values of x from 0° to 540°

(2)

13. Shown below is the graph of y = cos x



On the grid, sketch the graph of y = cos(x – 90°) for values of x from 0° to 360°

(2)

14. Describe the transformation that maps the curve with equation y = sin(x) onto the curve with equation

(a) y = –sin(x)

.....

.....

(2)

(b) y = 1 + sin(x)

.....

.....

(2)

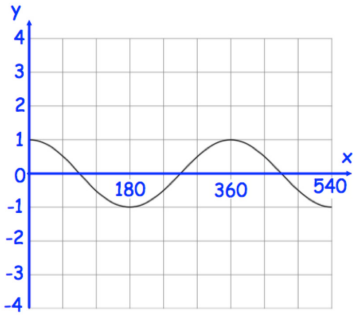
(c) y = sin(x – 30°)

.....

.....

(2)

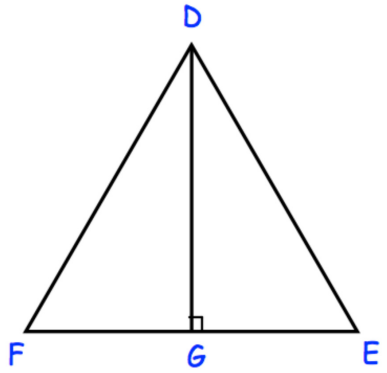
15. Shown below is the graph of y = cos x



On the grid, sketch the graph of y = 2 – cos(x) for values of x from 0° to 540°

(2)

9. DEF is an equilateral triangle.

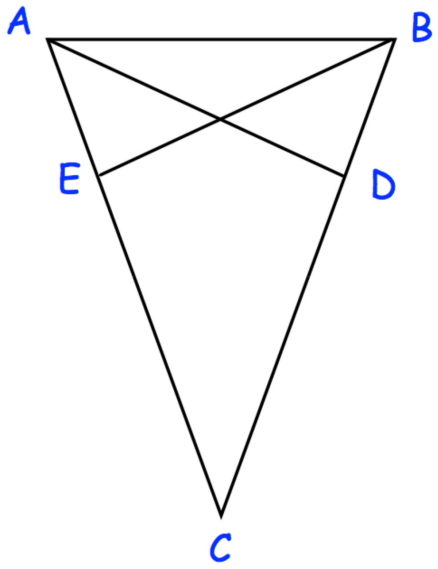


G lies on EF.
DG is perpendicular to FE.

Prove DFG is congruent to DEG.

(3)

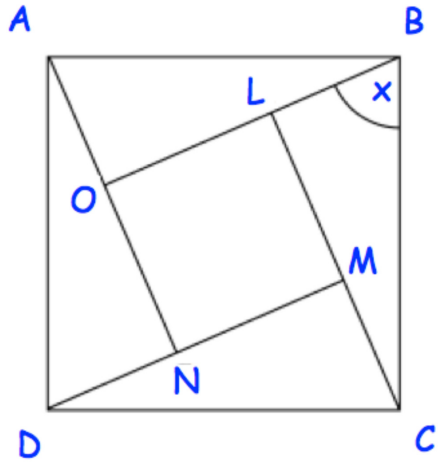
10. ABC is an isosceles triangle in which AC = BC.
D and E are points on BC and AC such that CE = CD.



Prove triangles ACD and BCE are congruent.

(4)

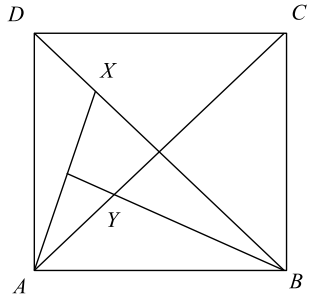
11. ABCD and LMNO are squares.
Angle CBL = x



Prove that triangles ABO and CBL are congruent.

(4)

12. ABCD is a square, X is a point in the diagonal BD and the perpendicular from B to AX meets AC in Y.



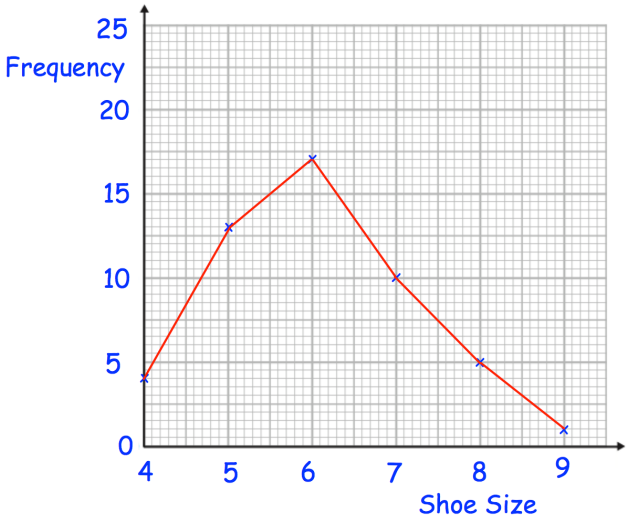
Prove that triangles AXD and AYB are congruent.

(4)

Frequency Polygons

Videos 155 and 156 on www.corbettmaths.com

Question 2: Henry surveyed 50 people.
This frequency polygon shows their shoe sizes.



(a) What is the modal shoe size?

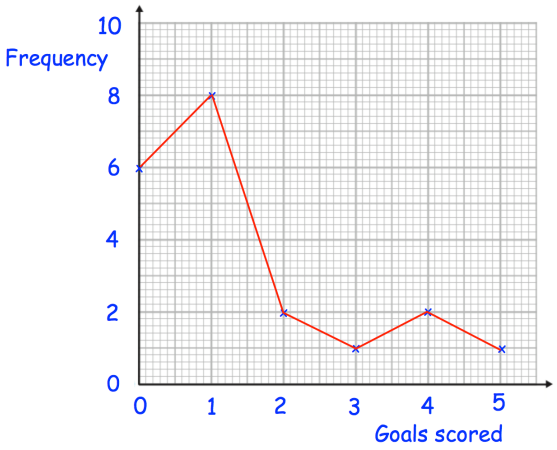
(b) What is the range of the shoe sizes?

(c) What fraction of the people surveyed have size 5 shoes?

(d) What percentage of the people surveyed have size 7 shoes?

(e) Henry picks somebody at random to win a prize.
Write down the probability that the winner has size 6 shoes.

Question 3: Roy is a striker for Rovers.
The frequency polygon shows the number of goals scored in each game over 20 games he has played.



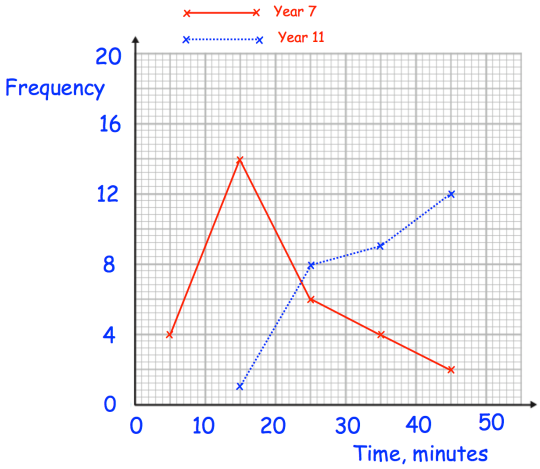
(a) Work out the median number of goals scored per game.

(b) Work out the mean number of goals scored per game.

(c) A journalist asks him for the "average" number of goals scored per game.
Which average should he use?

Question 4: The frequency polygons show the amount of time that 30 students in year 7 and 30 students in year 11 spent on their last maths homework.

Compare the time spent on homework by the year 7s and the year 11s.



Frequency Polygons

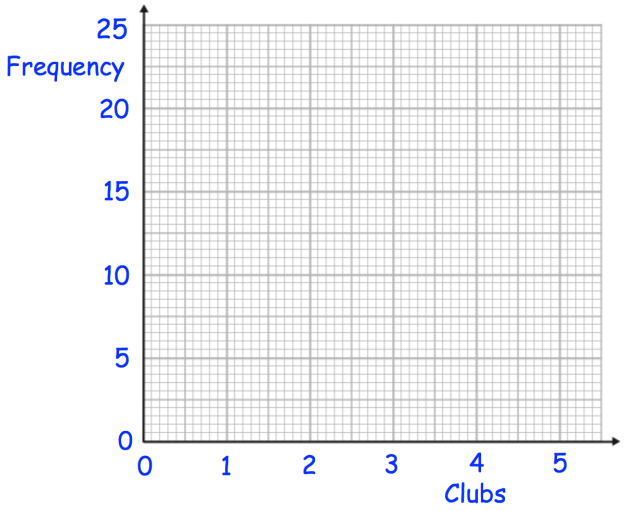
Videos 155 and 156 on www.corbettmaths.com

Question 5: 50 boys and 50 girls attend a primary school.
The table below shows how many clubs they attend.

(a) On the same grid, draw a frequency polygon for the boys and a frequency polygon for the girls.

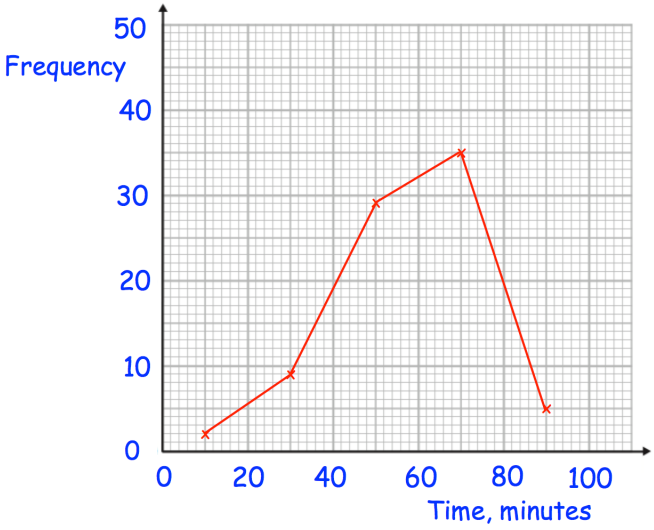
(b) Compare the distributions.

Clubs	Boys	Girls
0	5	2
1	20	18
2	14	22
3	9	7
4	2	1

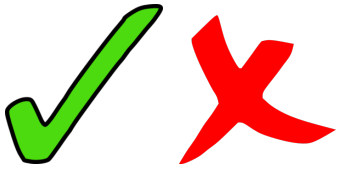


Question 6: The frequency polygon shows information about the amount of time people spend in the gym.

Calculate an estimate of the mean time spent in the gym.



Answers



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Error Intervals

Video 377 on www.corbettmaths.com

Examples

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Workout

Question 1: The mass of a coin is 8 grams to the nearest gram.
Complete the error interval for the mass of the coin

..... g ≤ mass < ..... g

Question 2: The distance between two cities is 900km to the nearest 100km.
Complete the error interval for the distance

..... km ≤ distance < ..... km

Question 3: Frank rounds a number, y, to the nearest ten.
His result is 20
Write down the error interval for y

Question 4: Lily rounds a number, y, to the nearest whole number.
Her result is 5
Write down the error interval for y

Question 5: Freya rounds a number, y, to one decimal place.
Her result is 6.4
Write down the error interval for y

Question 6: Oscar rounds a number, y, to the nearest integer.
His result is 100
Write down the error interval for y

Question 7: A number, n, is rounded to 1 decimal place.
The result is 1.3
Using inequalities, write down the error interval for n.

Question 8: A number, n, is rounded to 2 decimal places.
The result is 6.27
Using inequalities, write down the error interval for n.

Question 9: Elliott weighs 56.2kg.
This mass, m, is to the nearest 100g.
Write the error interval due to rounding.

Error Intervals

Video 377 on www.corbettmaths.com

Question 10: A number, x, is 21 when rounded to 2 significant figures.
Write down the error interval.

Question 11: A number, y, is 15000 when rounded to 2 significant figures.
Write down the error interval.

Question 12: A number, y, is 680000 when rounded to 3 significant figures.
Write down the error interval.

Question 13: The length of a line, l, was given as 2.8cm, truncated to 1 decimal place.
Complete the error interval for l

..... $cm \leq l <$ ..... cm

Question 14: A number, y, is 0.37 when truncated to 2 decimal places.
Complete the error interval for y

..... $\leq y <$ .....

Question 15: A number, n, is truncated to 1 decimal place.
The result is 18.1
Using inequalities, write down the error interval for n.

Question 16: A number, n, is truncated to 3 decimal places.
The result is 4.066
Using inequalities, write down the error interval for n.

Apply

Question 1: The length of each side of a regular hexagon is 4.7cm to 1 decimal place.
Write the error interval for the perimeter, P

Question 2: Grace and George complete a crossword.
It takes Grace 9 minutes to complete the crossword to the nearest minute.
It takes George 11 minutes to complete the crossword to the nearest minute.

Show that the total time for both people to complete the crossword could be 20 minutes 50 seconds.

Question 3: A man jogs 200 metres to the nearest 10 metres.
It takes him 30 seconds to the nearest 10 seconds.

Work out the error interval for his speed, s.

Error Intervals

Video 377 on www.corbettmaths.com

Question 4: A number, x, is 1.92 when truncated to 2 decimal places.
Matthew has been asked to write down the error interval.
This is his answer.

$$1.915 \leq x < 1.925$$

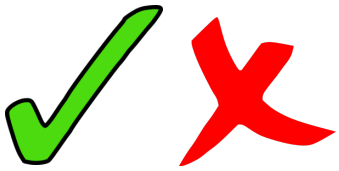
Explain why Matthew is wrong.

Question 5: A number, n, is rounded to 3 significant figures.
The result is 7500
Norris has been asked to write down the error interval for n.
This is his answer.

$$7450 < x < 7550$$

Explain why Norris is wrong.

Answers



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Corbett maths

Two Way Tables

Video 319 on www.corbettmaths.com

Apply

Question 1: Paul has a deck of 50 cards, each with a shape on it.
The shapes are either red or black.

	Square	Rectangle	Kite
Red	17	6	1
Black	4	9	13

Paul picks a card at random.

(a) What is the probability that the card has a black kite on it?
(b) What is the probability that the card has a red shape on it?
(c) What is the probability that the card has a square on it?
(d) What is the probability that the card has a shape with at least 2 lines of symmetry?

Question 2: 60 people visited a swimming pool one evening.
13 out of the 19 people who wore goggles were adults.
There were 15 children.

(a) Complete a two-way table for this information.
(b) How many adults did not were goggles?
(c) What fraction of the children wore goggles?

Question 3: 100 families booked a holiday in July or in August, at a travel agents.
Some of the families booked to go to France.
Some booked to go to Spain.
The rest of the families booked a holiday to Portugal.

59 families booked to go on holiday in August.
19 of the 35 families going to France booked to go in July.
30 families booked to go to Portugal.
20 families booked to go to Spain in August.

(a) Create a two-way table for this information.
(b) How many families booked to go to Portugal in July?

Question 4: There are 120 students in Year 11 at a school.
Each student studies one language, either French, Spanish, German or Welsh.
21 of the 40 students studying Welsh are male.
18 males and 9 females study French.
12 of the 17 students studying Spanish are female.
Twice as many females study German than males.

How many students in Year 11 are female?

Two Way Tables

Video 319 on www.corbettmaths.com

Question 5: A teacher surveys 64 children on how they travelled to school.
20 of the students were in Year 7.
The teacher surveyed 30% more students in Year 9 than in Year 7.
The rest of the students surveyed were in Year 11.
75% of the students in Year 7 walked to school.
8 more students in Year 9 walked to school than did not walk.
Out of students surveyed, more Year 11 students walked to school than Year 9 students.

One of these students is picked at random

Write down the probability that the student chosen will walk to school.

Question 6: Isla has a box of counters.
The table shows information about the shape and colours of the counters.

		Shape		
		Circle	Triangle	Square
Colour	Blue	6	2	5
	Red	8	9	11

Isla picks a counter at random, looked at it and then returned it to the box.

(a) Given it is a circular counter, what is the probability that it was red?

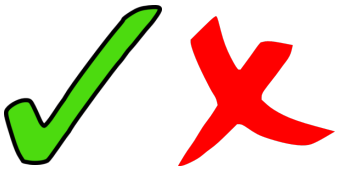
David picks a counter at random, looked at it and then returned it to the box.

(b) Given it is a blue counter, what is the probability that it was triangular?

Emily adds a number of red square counters to the box.
The probability of Emily picking a red square at random is now $\frac{2}{3}$

(c) How many red square counters did Emily add to the box?

Answers



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Box Plots

Videos 149 and 150 on www.corbettmaths.com

Question 4: Draw a box plot for each set of data

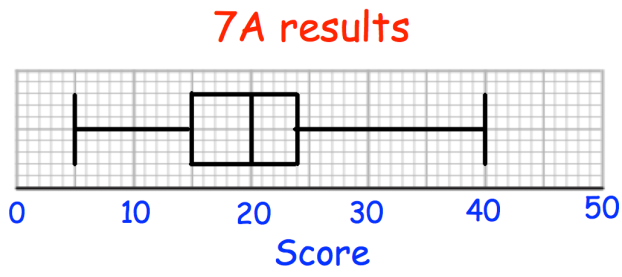
(a) 8, 10, 13, 14, 14, 15, 15, 16, 18, 19, 21, 22, 24, 29, 35

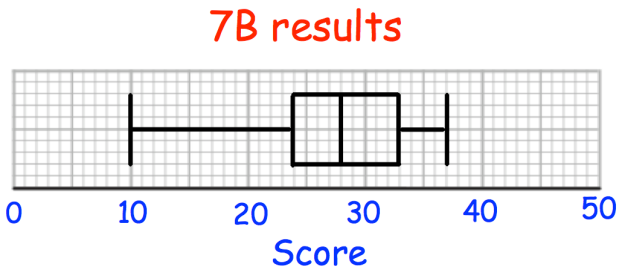
(b) 40, 80, 90, 90, 100, 120, 130

(c) 5.9, 7.3, 7.8, 8, 8.4, 8.7, 8.9, 8.9, 8.9, 9, 9, 9.1, 9.1, 9.3, 9.5, 9.6, 9.9, 10.5, 10.9

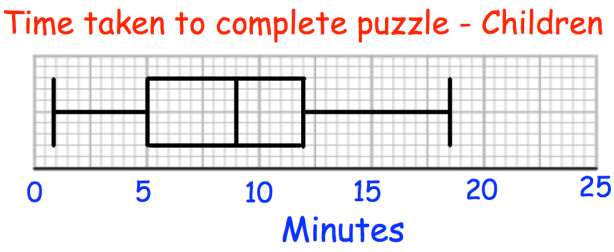
Question 5: Compare the distributions of each pair of box plots below.

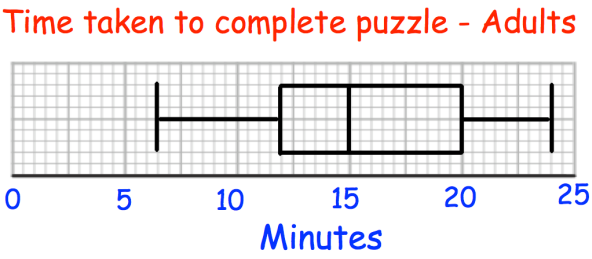
(a)



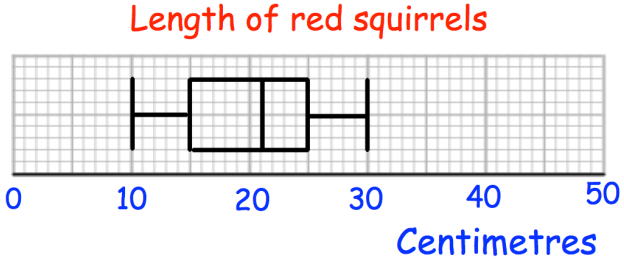


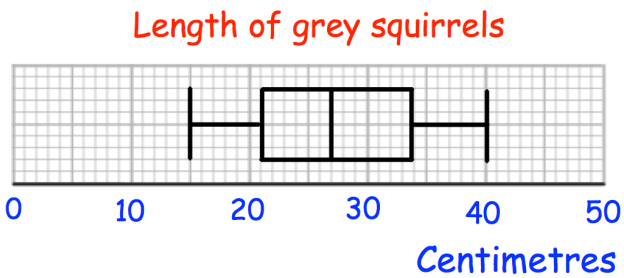
(b)



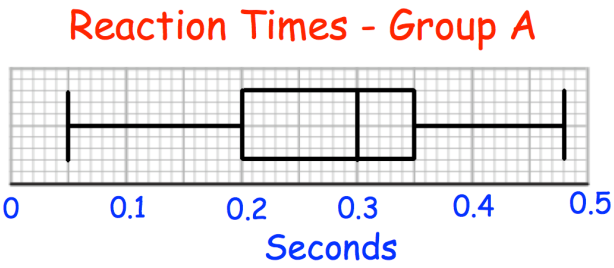


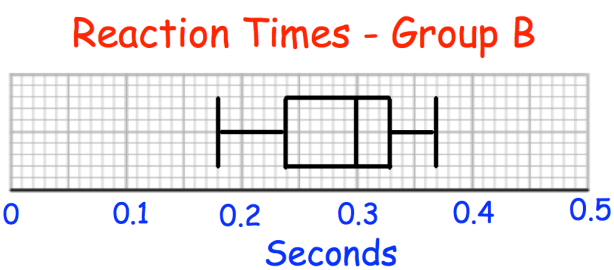
(c)





(d)





Box Plots

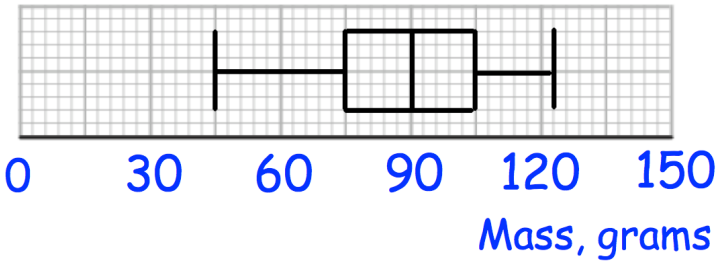
Videos 149 and 150 on www.corbettmaths.com

Question 3: Mr Jones is an estate agent on the Isle of Man.
He has created this table to show information about the prices of houses he has sold.

Explain how you know he has made a mistake.

Median	£375,000
Range	£235,000
Interquartile Range	£590,000

Question 4: The box plot show information about the masses of apples in a crate.



Jack is going to select apples at random from the crate.
After selecting each apple, he records its mass and returns it to the crate before picking another.
Work out the probability that:

(a) Jack picks two apples, both under 75g

(b) Jack picks two apples, both over 90g

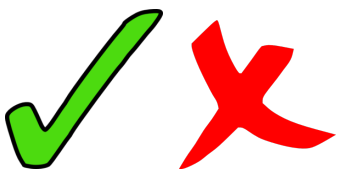
(c) Jack picks two apples, both over 105g

(d) Jack picks two apples, one under 90g and one over 105g

(e) Jack picks three apples, all over 105g

(f) Jack picks three apples, two over 105g and one under 75g.

Answers



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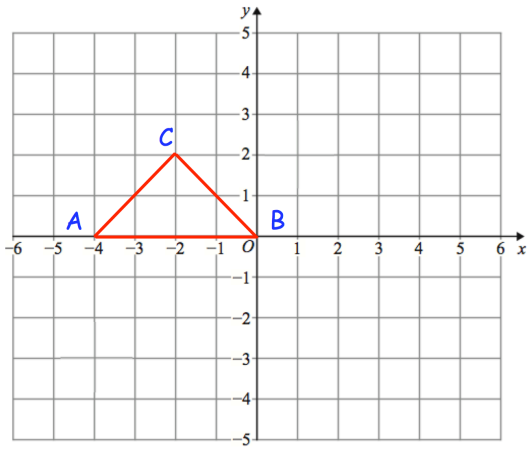
Invariant Points

Video 392 on www.corbettmaths.com

Apply



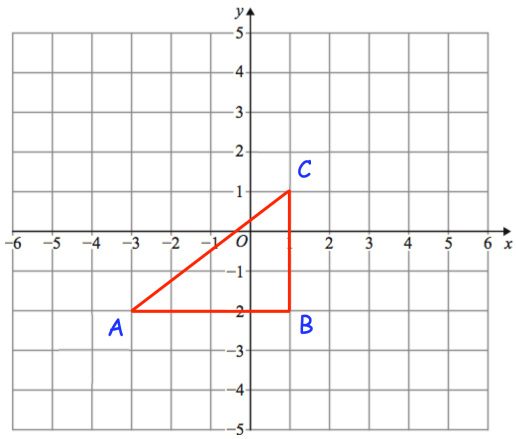
Question 1: ABC is a triangle.

Describe fully a **single** transformation of ABC so that:

(a) None of the vertices are invariant.

(b) Exactly one vertex is invariant.

(c) Exactly two vertices are invariant.



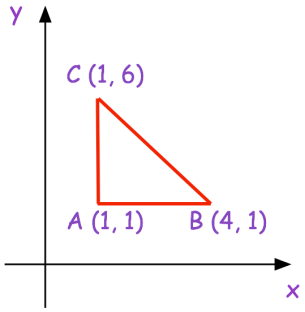
Question 2: Here is triangle ABC

Olivia says "if ABC is reflected in the line $x = -3$ there is one invariant point."

Amelia says "if ABC is reflected in the line $y = -2$ there are two invariant points."

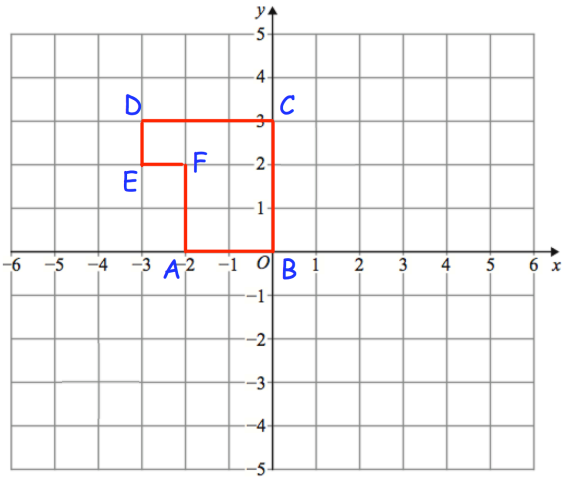
Isla says "if ABC is reflected in the line $x = 1$ there are two vertices that are invariant."

Which student is incorrect? Explain your answer.



Question 3: Here is a sketch of triangle ABC.

Describe fully a **single** transformation of ABC so that all the points on AC are invariant and the point B is not invariant.



Question 4: Here is shape ABCDEF

Describe fully **single** transformations so that from the six vertices:

(a) only vertices B and C are invariant.

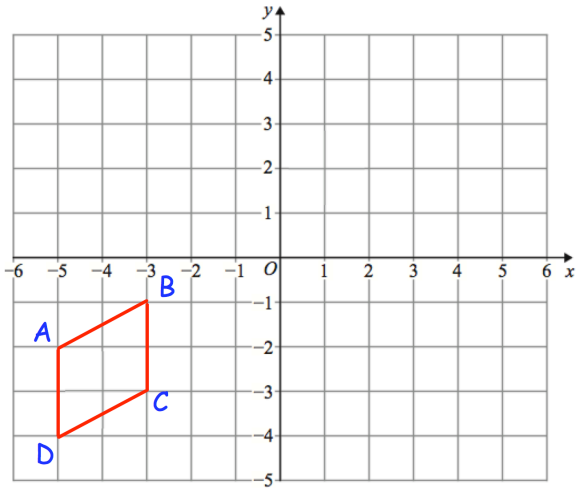
(b) only vertex F is invariant.

(c) only vertices B, D and F are invariant.

Invariant Points

Video 392 on www.corbettmaths.com

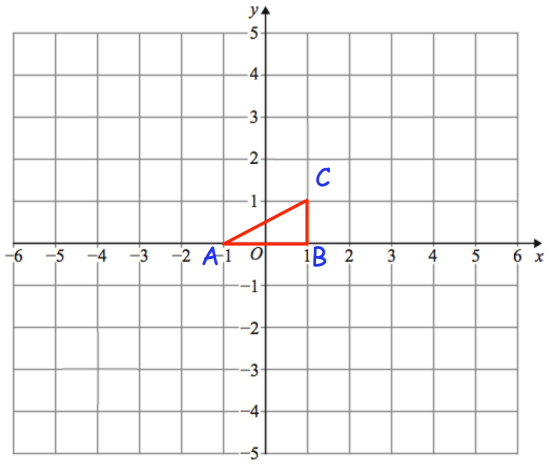
Question 5: Here is quadrilateral ABCD



ABCD is reflected in the line $x = -1$
followed by a reflection in the line $y = -x$
followed by a rotation of 180° about $(-1, -1)$

Which of the vertices are invariant?

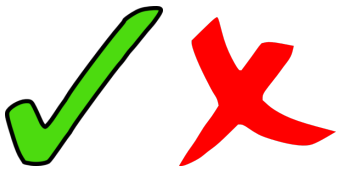
Question 6: Shown is triangle ABC



ABC is rotated 180° about $(-1, 2)$ and then translated by the vector $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$

Write down the coordinate of the invariant point.

Answers



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