E/P **8** A curve has the equation $y = \sin 5x + \cos 3x$. Find the equation of the tangent to the curve at the point $(\pi, -1)$. **(4 marks)**

E/P **9** A curve has the equation $y = 2x^2 - \sin x$. Show that the equation of the normal to the curve at the point with x-coordinate π is

$$x + (4\pi + 1)y - \pi(8\pi^2 + 2\pi + 1) = 0$$ **(7 marks)**

E/P **9** Prove that the derivative of a^{kx} is $a^x k \ln a$. You may assume that the derivative of e^{kx} is ke^{kx}. **(4 marks)**

E/P **10** $f(x) = e^{2x} - \ln(x^2) + 4, x > 0$

a Find $f'(x)$. **(3 marks)**

The curve with equation $y = f(x)$ has a gradient of 2 at point P. The x-coordinate of P is a.

b Show that $a(e^{2a} - 1) = 2$. **(2 marks)**

E/P **11** A curve C has equation

$$y = 5\sin 3x + 2\cos 3x, -\pi \leqslant x \leqslant \pi$$

a Show that the point $P(0, 2)$ lies on C. **(1 mark)**

b Find an equation of the normal to the curve C at P. **(5 marks)**

E/P **12** The point P lies on the curve with equation $y = 2(3^{4x})$. The x-coordinate of P is 1. Find an equation of the normal to the curve at the point P in the form $y = ax + b$, where a and b are constants to be found in exact form. **(5 marks)**

E/P **10** The curve C has equation $x = 4\cos 2y$.

a Show that the point $Q\left(2, \frac{\pi}{6}\right)$ lies on C. **(1 mark)**

b Show that $\frac{dy}{dx} = -\frac{1}{4\sqrt{3}}$ at Q. **(4 marks)**

c Find an equation of the normal to C at Q. Give your answer in the form $ax + bx + c = 0$, where a, b and c are exact constants. **(4 marks)**

E/P **12** The curve C has equation $y = \frac{4}{(2 - 4x)^2}, x \neq \frac{1}{2}$

The point A on C has x-coordinate 3.

Find an equation of the normal to C at A in the form $ax + by + c = 0$, where a, b and c are integers. **(7 marks)**

E/P **13** Find the exact value of the gradient of the curve with equation $y = 3^{x^3}$ at the point with coordinates $(1, 3)$. **(4 marks)**

E/P **6** A curve C has equation $y = x^2 \cos(x^2)$. Find the equation of the tangent to the curve C at the point $P\left(\frac{\sqrt{\pi}}{2}, \frac{\pi\sqrt{2}}{8}\right)$ in the form $ax + by + c = 0$ where a, b and c are exact constants. **(7 marks)**

E/P **7** Given that $y = 3x^2(5x - 3)^3$, show that

$$\frac{dy}{dx} = Ax(5x - 3)^n(Bx + C)$$

where n, A, B and C are constants to be determined. **(4 marks)**

E/P **8** The curve C has equation $x = \frac{e^y}{3 + 2y}$

a Find the coordinates of the point P where the curve cuts the x-axis. **(1 mark)**

b Find an equation of the normal to the curve at P, giving your answer in the form $y = mx + c$, where m and c are integers to be found. **(6 marks)**

E/P **10** A curve C has equation $y = \frac{e^{2x}}{(x - 2)^2}$, $x \neq 2$.

a Show that

$$\frac{dy}{dx} = \frac{Ae^{2x}(Bx - C)}{(x - 2)^3}$$

where A, B and C are integers to be found. **(4 marks)**

b Find the equation of the tangent of C at the point $x = 1$. **(3 marks)**

E/P **11** Given that

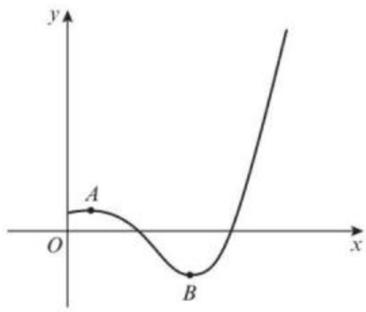
$$f(x) = \frac{2x}{x + 5} + \frac{6x}{x^2 + 7x + 10}, x > 0$$

a show that $f(x) = \frac{2x}{x + 2}$ **(4 marks)**

b Hence find $f'(3)$. **(3 marks)**

E/P **12** The diagram shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = \frac{2\cos 2x}{e^{2-x}}, 0 < x < \pi$$



The curve has a maximum turning point at A and a minimum turning point and B as shown in the diagram.

a Show that the x-coordinates of point A and point B are solutions to the equation $\tan 2x = \frac{1}{2}$ **(4 marks)**

b Find the range of $f(x)$. **(2 marks)**

E/P **5** The curve C has equation

$$y = \frac{1}{\cos x \sin x}, 0 < x \leqslant \pi$$

a Find $\frac{dy}{dx}$ **(4 marks)**

b Determine the number of stationary points of the curve C. **(2 marks)**

c Find the equation of the tangent at the point where $x = \frac{\pi}{3}$, giving your answer in the form $ax + by + c = 0$, where a, b and c are exact constants to be determined. **(3 marks)**

E/P **11** **a** Curve C has equation $x = (\arccos y)^2$. Show that

$$\frac{dy}{dx} = -\frac{\sqrt{1 - \cos^2 \sqrt{x}}}{2\sqrt{x}}$$ **(5 marks)**

E/P **12** Given that $x = \operatorname{cosec} 5y$,

a find $\frac{dy}{dx}$ in terms of y. **(2 marks)**

b Hence find $\frac{dy}{dx}$ in terms of x. **(4 marks)**

E/P **7** A curve has parametric equations

$$x = 2\sin^2 t, \quad y = 2\cot t, \quad 0 < t < \frac{\pi}{2}$$

a Find an expression for $\frac{dy}{dx}$ in terms of the parameter t. **(4 marks)**

b Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{6}$ **(4 marks)**

E/P **8** The curve C has parametric equations

$$x = 4\sin t, \qquad y = 2\operatorname{cosec} 2t, \qquad 0 \leqslant t \leqslant \pi$$

The point A lies on C and has coordinates $\left(2\sqrt{3}, \frac{4\sqrt{3}}{3}\right)$.

a Find the value of t at the point A. **(2 marks)**

The line l is a normal to C at A.

b Show that an equation for l is $9x - 6y - 10\sqrt{3} = 0$. **(6 marks)**

E/P **9** The curve C has parametric equations

$$x = t^2 + t, \quad y = t^2 - 10t + 5, \quad t \in \mathbb{R}$$

where t is a parameter. Given that at point P, the gradient of C is 2,

a find the coordinates of P **(4 marks)**

b find the equation of the tangent to C at point P **(3 marks)**

c show that the tangent to C at point P does not intersect the curve again. **(5 marks)**

Problem-solving

Substitute the equations for x and y into the equation of your tangent, and show that the resulting quadratic equation has no real roots.

(E/P) **10** The curve C has parametric equations

$$x = 2\sin t, \quad y = \sqrt{2}\cos 2t, \quad 0 < t < \pi$$

a Find an expression for $\frac{dy}{dx}$ in terms of t. **(2 marks)**

The point A lies on C where $t = \frac{\pi}{3}$. The line l is the normal to C at A.

b Find an equation for l in the form $ax + by + c = 0$, where a, b and c are exact constants to be found. **(5 marks)**

c Prove that the line l does not intersect the curve anywhere other than at point A. **(6 marks)**

(E/P) **11** A curve has parametric equations

$$x = \cos t, \quad y = \tfrac{1}{2}\sin 2t, \quad 0 \leqslant t < 2\pi$$

a Find an expression for $\frac{dy}{dx}$ in terms of t. **(2 marks)**

b Find an equation of the tangent to the curve at point A where $t = \frac{\pi}{6}$ **(4 marks)**

The lines l_1 and l_2 are two further distinct tangents to the curve. Given that l_1 and l_2 are both parallel to the tangent to the curve at point A,

c find an equation of l_1 and an equation of l_2 **(6 marks)**

(E/P) **7** A curve C is described by the equation

$$2x^2 + 3y^2 - x + 6xy + 5 = 0$$

Find an equation of the tangent to C at the point $(1, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(7 marks)**

(E/P) **8** A curve C has equation

$$3^x = y - 2xy$$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates $(2, -3)$. **(7 marks)**

(E/P) **9** Find the gradient of the curve with equation

$$\ln(y^2) = \tfrac{1}{2}x\ln(x - 1), \quad x > 1, \quad y > 0$$

at the point on the curve where $x = 4$. Give your answer as an exact value. **(7 marks)**

(E/P) **10** A curve C satisfies $\sin x + \cos y = 0.5$, where $-\pi < x < \pi$ and $-\pi < y < \pi$.

a Find an expression for $\frac{dy}{dx}$ **(2 marks)**

b Find the coordinates of the stationary points on C. **(5 marks)**

(E/P) **11** The curve C has the equation $ye^{-3x} - 3x = y^2$.

a Find $\frac{dy}{dx}$ in terms of x and y. **(5 marks)**

b Show that the equation of the tangent to C at the origin, O, is $y = 3x$. **(4 marks)**

(E/P) **11** A curve C has equation

$$y = \tfrac{1}{3}x^2\ln x - 2x + 5, \; x > 0$$

Show that the curve C is convex for all $x \geqslant e^{-\frac{3}{2}}$. **(5 marks)**