

**GCE AS Mathematics (8MA0) – Paper 1
Pure Mathematics**

Summer 2018 student-friendly mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn't show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.

Guidance on the use of codes within this document

M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.

A1 – accuracy mark. This mark is generally given for a correct answer following correct working.

B1 – working mark. This mark is usually given when working and the answer cannot easily be separated.

Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

Question 1 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
		M1	This mark is given for an attempt to integrate (for any correct power)
	$\frac{2}{3} \times \frac{x^4}{4} + \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + x$	A1	This mark is given for finding $\frac{x^4}{6}$ (may be unsimplified at this stage) and x
		A1	This mark is given for all three terms (may be unsimplified at this stage)
	$\frac{x^4}{6} - 4x^{\frac{3}{2}} + x + c$	A1	This mark is given for a completely correct, simplified expression, including the constant of integration.

Question 2 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(i)	$x^2 - 8x + 17 = (x - 4)^2 - 16 + 17$	M1	This mark is given for an attempt to complete the square
	$= (x - 4)^2 + 1$	A1	This mark is given for completing the square correctly
	$(x - 4)^2 \geq 0,$ hence $x^2 - 8x + 17 > 0$ for all x	A1	This mark is given for a complete explanation.
(ii)	For example, $x = -5$: $(-5 + 3)^2 = 4$, whereas $(-5)^2 = 25$	M1	This mark is given for an example showing that it may not always be true
	Sometimes true. For example: For $x = -5$, $(-5 + 3)^2 = 4$, whereas $(-5)^2 = 25$ (False) For $x = 5$ $(5 + 3)^2 = 64$, whereas $5^2 = 25$ (True)	A1	This mark is given for stating that it is sometimes true, giving reasons

Question 3 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\vec{AB} = \vec{OB} - \vec{OA}$	M1	This mark is given for an attempt at subtraction either way around (this may be implied by one correct component)
	$\vec{AB} = (-5\mathbf{i} - 2\mathbf{j}) - (4\mathbf{i} - 5\mathbf{j})$ $= -9\mathbf{i} + 3\mathbf{j}$	M1	This mark is given for the correct answer only (allowing for column notation)
(b)	$ AB = \sqrt{((-9)^2 + 3^2)}$	A1	This mark is given for the correct use of Pythagoras theorem or modulus formula
	$= 3\sqrt{10}$	A1	This mark is given for the correct answer as a simplified surd only

Question 4 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	Gradient of l_1 is $\frac{3}{4}$	B1	This mark is given for finding the gradient of l_1
	Gradient of l_2 is $\frac{-1-8}{5-(-1)}$	M1	This mark is given for an attempt to find the gradient of l_2 , the line joining (5, -1) and (-1, 8)
	$= -\frac{3}{2}$	A1	This mark is given for finding the gradient of l_2
	Lines l_1 and l_2 are neither parallel (since the gradients are not equal) nor perpendicular (since $\frac{3}{4} \times -\frac{3}{2} \neq -1$)	A1	This mark is given for stating a correct conclusion with suitable reasons given

Question 5 (Total 5 marks)

Part	Working or answer an examiner might expect to see		Mark	Notes
(a)	Between lines 1 and 2: “Student cannot use the subtraction law without dealing with the 2” or “Line 2 should be $\log_2 \left(\frac{x^2}{\sqrt{x}} \right)$		B1	This mark is awarded for finding one error
	Between lines 4 and 5: “Student undoes the logs incorrectly” or “Line 5 should be $x = 2^3 = 8$ ”		B1	This mark is awarded for finding a second error
(b)	Version 1	Version 2		
	$\log_2 \left(\frac{x^2}{\sqrt{x}} \right) = 3$	$\frac{3}{2} \log_2 x = 3$	M1	This mark is given for a correct method to combine the log terms
	$x^{\frac{3}{2}} = 2^3$	$x = 2^2$	M1	This mark is given for correct index work to give an expression in the form $Ax^n = 2^b$
	$x = (2^3)^{\frac{2}{3}} = 4$	$x = 4$	A1	This mark is given for a correct answer with all steps shown

Question 6 (Total 7 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$P = 100 - 6.25(15 - 9)^2$	M1	This mark is given for substituting a value of 15 into the equation
	$= -125$ This is not a sensible selling price because the company would make an annual loss of £125,000	A1	This mark is given for finding -125 and a complete explanation
(b)	$P > 80 \Rightarrow (x - 9)^2 < \frac{6.25}{20} = 3.2$	M1	This mark is given for finding an expression in the form $(x - 9)^2 < k$
	$9 - \sqrt{3.2} < x < 9 + \sqrt{3.2}$	M1	This mark is given for solving to find two possible positive values for x
	$x > 7.21$, so minimum price = £7.22	A1	This mark is given for correctly finding the minimum price
(c)	Maximum $P = 100,000$	B1	This mark is given for finding the maximum possible profit of the company
	Maximum profit when $x = 9$	B1	This mark is given for the corresponding selling price of the toy

Question 7 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$15 = \frac{1}{2} \times 5 \times 10 \sin \theta$	M1	This mark is given for using the equation for the area of a triangle to find $\sin \theta$
	$\sin \theta = \frac{3}{5}$	A1	This mark is given for finding the value of $\sin \theta$
	$\cos^2 \theta = 1 - \sin^2 \theta$ $\Rightarrow \cos^2 \theta = 1 - \frac{9}{25}$	M1	This mark is given for using a valid method to find the values of $\cos \theta$
	$\cos \theta = \pm \frac{4}{5}$	A1	This mark is given for finding two values of $\cos \theta$
	$BC^2 = 10^2 + 5^2 - 2 \times 10 \times 5 \times \frac{4}{5}$	M1	This mark is given for using the cosine rule to find a value for the length of BC
	$BC = \sqrt{205}$	M1	This mark is given for finding the exact value of the length of BC

Question 8 (Total 9 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)(i)	$C = 1500v^{-1} + \frac{2}{11}v + 60$	M1	This mark is given for an attempt to differentiate (dealing with powers of v correctly)
	$\frac{dC}{dv} = -\frac{1500}{v^2} + \frac{2}{11}$	A1	This mark is given for differentiating correctly
	$\frac{dC}{dv} = 0 \Rightarrow v^2 = 8250$	M1	This mark is given for setting $\frac{dC}{dv} = 0$ and rearranging to find a value for v^2
	$v = \sqrt{8250} = 90.8 \text{ kmh}^{-1}$	A1	This mark is given for finding a correct value of v
(a)(ii)	$C = \frac{1500}{908} + \frac{2}{11} + 60$	M1	This mark is given for substitution of 90.8 into the equation for the model
	$= \text{£}93.03$	A1	This mark is given for finding the cost of the journey (accept answers which round to £93)
(b)	$\frac{d^2C}{dv^2} = +\frac{3000}{v^3}$	M1	This mark is given for finding the second derivative of C
	At $v = 90.8$, $\frac{d^2C}{dv^2} = 0.004$ $0.004 > 0$, hence it is a minimum cost	A1	This mark is given for a correct calculation, statement and conclusion.
(c)	For example: It would be impossible to drive at this speed over the whole journey It would be impossible to drive at a constant speed over the whole journey	B1	This mark is given for stating a sensible limitation of the model

Question 9 (Total 9 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$gf(-2) = (4 \times -8) - (12 \times 4) - (15 \times -2) + 50$	M1	This mark is given for an attempt to find $gf(-2)$
	$gf(-2) = 0$, thus $(x + 2)$ is a factor	A1	This mark is given for finding that $gf(-2) = 0$ and stating a correct conclusion
(b)	$4x^3 - 12x^2 - 15x + 50$	M1	This mark is given for an attempt to divide $4x^3 - 12x^2 - 15x + 50$ by $(x + 2)$
	$= (x + 2)(4x^2 - 20x + 25)$	A1	This mark is given for correctly dividing $4x^3 - 12x^2 - 15x + 50$ by $(x + 2)$
	$= (x + 2)(2x \pm \dots)(2x \pm \dots)$	M1	This mark is given for an attempt to factorise $(4x^2 - 20x + 25)$
	$= (x + 2)(2x - 5)^2$	A1	This mark is given for a correct answer
(c)(i)	To find x when $(x + 2)(2x - 5)^2 \leq 0$	M1	This mark is given for deducing that the solution will be where the curve is on or below the axis
	$x \leq -2$ or $x = 2.5$	A1	This mark is given for two correct solutions
(c)(ii)	Since $gf(-2) = 0$ and $gf(2.5) = 0$, $gf(2x) = 0 \Rightarrow x = -1$ or $x = 1.25$	B1	This mark is given for deducing that $gf(2x) = 0$ at the values shown

Question 10 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\frac{(x+h)^3 - x^3}{h}$	B1	This mark is given for sight of $\frac{(x+h)^3 - x^3}{h}$
	$(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$	M1	This mark is given for a correct expansion of $(x + h)^3$
	Thus gradient = $\frac{3x^2h + 3xh^2 + h^3}{h}$ $= 3x^2 + 3xh + h^2$	M1	This mark is given for substituting and simplifying
	As $h \rightarrow 0$, gradient $\rightarrow 3x^2$ Thus $\frac{d}{dx}x^3 = 3x^2$	A1	This mark is given for a complete proof with a consideration of the limit

Question 11 (Total 8 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\left(2 - \frac{x}{16}\right)^9$ $= 2^9 + \binom{9}{1} \times 2^8 + \binom{9}{2} \times 2^7 \times \left(-\frac{x}{16}\right)^2 + \dots$	M1	This mark is given for an attempt at a binomial expansion
	$\left(2 - \frac{x}{16}\right)^9 = 512 + \dots$	B1	This mark is given for evaluating the first term
	$\dots -144x + \dots$	A1	This mark is given for evaluating the second term
	$+ 18x^2 + \dots$	A1	This mark is given for evaluating the third term
(b)	$512a = 128$	M1	This mark is given for an attempt to find a
	$a = \frac{1}{4}$	A1	This mark is given for the correct answer only
(c)	$512b - 144a = 36$	M1	This mark is given for an attempt to find b
	$b = \frac{9}{64}$	A1	This mark is given for the correct answer only

Question 12 (Total 8 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$4 \cos \theta - 1 = 2 \sin \theta \times \frac{\sin \theta}{\cos \theta}$	M1	This mark is given for recall and use of the identity $\tan \theta = \frac{\sin \theta}{\cos \theta}$
	$4 \cos^2 \theta - \cos \theta = 2 \sin^2 \theta$	M1	This mark is given for multiplying both sides by $\cos \theta$
	$4 \cos^2 \theta - \cos \theta = 2(1 - \cos^2 \theta)$	M1	This mark is given for use of the identity $1 - \cos^2 \theta = \sin^2 \theta$
	$6 \cos^2 \theta - \cos \theta - 2 = 0$	A1	This mark is given for fully correct reasoning with no errors
(b)	Set $\cos 3x = a$ and $6a^2 - a - 2 = 0$ $(3a - 2)(2a + 1) = 0$	M1	This mark is given for attempting to solve the quadratic in $\cos 3x$
	$\cos 3x = \frac{2}{3}, -\frac{1}{2}$	B1	This mark is given for finding two values for $\cos 3x$
	$x = \frac{1}{3} \arccos \frac{2}{3}$ or $\frac{1}{3} \arccos -\frac{1}{2}$	M1	This mark is given for expressions for x
	$x = 16.1^\circ, 40^\circ, 80^\circ$	A1	This mark is given for three correct answers only

Question 13 (Total 8 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$V = 10^{\frac{1}{20}t+4.8}$	M1	This mark is awarded for a method to find V
	$V = 10^{\frac{1}{20}t} + 10^{4.8}$	M1	This mark is awarded for a method for forming an equation in the form $V = pq^t$
	$p = 10^{4.8} = 63100$	A1	This mark is given for an evaluation of p to 4 significant figures
	$q = 10^{\frac{1}{20}} = 1.222$	A1	This mark is given for an evaluation of q to 4 significant figures
(b)	The value of the painting on 1 January 1980	B1	This mark is given for a correct statement
	The proportional increase in the value of the painting each year	B1	This mark is given for a correct statement
(c)	$V = 63100 \times 1.222^{30}$	M1	This mark is given for finding an expression for V
	$= \text{£}1.99 \text{ million}$	A1	This mark is given for a correct answer only

Question 14 (Total 9 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)(i)	$(x - 3)^2 + (y + 5)^2 - 25 = 0$	M1	This mark is given for attempting to complete the square
	Centre $(3, -5)$	A1	This mark is given for find the coordinates of the centre of the circle
(a)(ii)	Radius 5	A1	This mark is given for finding the radius of the circle
(b)	$k < 0$	B1	This mark is given for deducing (by a sketch or otherwise) that $k < 0$
	$x^2 + kx^2 - 6x + 10kx + 9 = 0$	M1	This mark is given for an attempt to substitute $y = kx$ into the equation for C
	$(1 + k)^2x^2 + (10k - 6)x + 9 = 0$	A1	This mark is given forming a quadratic in x
	For $b^2 - 4ac > 0$, $(10k - 6)^2 - 4 \times (1 + k^2) \times 9 > 0$ $(10k - 6)^2 - 36(1 + k^2) > 0$	M1	This mark is given for an attempt to find k equation with has two distinct roots
	$(10k - 6)^2 - 36(1 + k^2) = 0$ for $k = 0, \frac{15}{8}$	M1	This mark is given for solving to find critical values
	$k < 0, k > \frac{15}{8}$	A1	This mark is given for finding the range of values for k

Question 15 (Total 10 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\frac{dy}{dx} =$	M1	This mark is given for an attempt to differentiate the equation for the curve C
	$\frac{dy}{dx} = -\frac{64}{x^3} + 3$	A1	This mark is given for correctly differentiating y
	$\frac{dy}{dx}$ at $x = 4 = 2$	M1	This mark is given for substituting $x = 4$ into $\frac{dy}{dx}$
	$y - 6 = -\frac{1}{2}(x - 4)$	M1	This mark is given for finding an equation of the normal
	The normal cuts the x -axis at $x = 16$	A1	This mark is given for substituting $y = 0$ to find where the normal at $(4, 6)$ cuts the x -axis
	$\frac{1}{2}(6 \times 12) = 36$	M1	This mark is given for finding the triangular area under l from $x = 4$ to $x = 16$
	$\int \frac{32}{x^2} + 3x - 8 \, dx = -\frac{32}{x} + \frac{3}{2}x^2 - 8x$	M1	This mark is given attempting to integrate the equation for the curve C
	$= -\frac{32}{x} + \frac{3}{2}x^2 - 8x$	A1	This mark is given for correctly integrating the equation for the curve C
	$\left[-\frac{32}{x} + \frac{3}{2}x^2 - 8x \right]_2^4 = -16 + 26 = 10$	M1	This mark is given for finding the area under the curve
	$10 + 36 = 46$	A1	This mark is given for showing the total area of the region R is 46