

Question 4

Carry out the following integrations:

1.
$$\int \frac{\cos 2x}{1 - \cos^2 2x} dx = -\frac{1}{2} \operatorname{cosec} 2x + C$$

2.
$$\int \cot^2 3x dx = -x - \frac{1}{3} \cot 3x + C$$

3.
$$\int \sin 2x \sec x dx = -2 \cos x + C$$

4.
$$\int \frac{1}{\sin x \cos^2 x} dx = \ln \left| \tan \left(\frac{x}{2} \right) \right| + \sec x + C$$

5.
$$\int \frac{1}{\sec x - 1} dx = -x - \cot x - \operatorname{cosec} x + C$$

6.
$$\int 1 - \cot^2 x dx = 2x + \cot x + C$$

7.
$$\int (2 \cos x - 3)^2 dx = 11x + \sin 2x - 12 \sin x + C$$

8.
$$\int (3 \sin x - \cos x)^2 dx = 5x - 2 \sin 2x + \frac{3}{2} \cos 2x + C = 5x - 2 \sin 2x - 3 \sin^2 x + C$$

9.
$$\int \frac{1}{\cos x \sin^2 x} dx = \ln |\sec x + \tan x| - \operatorname{cosec} x + C$$

10.
$$\int \sin^2 x \sec^2 x dx = \tan x - x + C$$

$$\begin{aligned}
 1. \int \frac{\cos 2x}{1-\cos 2x} dx &= \int \frac{\cos 2x}{\sin^2 x} dx = \int \frac{\cos 2x}{\sin 2x} \times \frac{1}{\sin x} dx \\
 &= \int \cot 2x \operatorname{cosec} x dx = -\frac{1}{2} \operatorname{cosec} 2x + C // \\
 2. \int \cot^2 2x dx &= \int \operatorname{cosec}^2 2x - 1 dx = -\frac{1}{2} \cot 2x - x + C // \\
 3. \int \sin 2x \operatorname{sech} x dx &= \int (2 \sin x \cos x) \frac{1}{\cosh x} dx = \int 2 \sin x dx = -2 \cos x + C // \\
 4. \int \frac{1}{\sin x \cosh x} dx &= \int \frac{\frac{\sin x}{\cos x}}{\frac{\cosh x}{\sinh x}} dx = \int \frac{1 + \tanh x}{\sin x} dx = \int \frac{1}{\sin x} + \frac{\tanh x}{\sin x} dx \\
 &= \int \operatorname{cosec} x + \frac{\sin^2 x}{\cos^2 x} \times \frac{1}{\sin x} dx = \int \operatorname{cosec} x + \frac{\sin x}{\cos^2 x} dx \\
 &= \int \operatorname{cosec} x + \frac{\sin x}{\cos^2 x} dx = \int \operatorname{cosec} x + \tan x \operatorname{sech} x dx \\
 &= \ln |\tan x| + \operatorname{sech} x + C // \\
 5. \int \frac{1}{\sec x - 1} dx &= \int \frac{\sec x + 1}{(\sec x - 1)(\sec x + 1)} dx = \int \frac{\sec x + 1}{\sec^2 x - 1} dx \\
 &= \int \frac{\sec x + 1}{\tan^2 x} dx = \int \frac{\sec x}{\tan^2 x} + \frac{1}{\tan^2 x} dx \\
 &= \int \frac{1}{\cos x} \cot^2 x + \cot^2 x dx = \int \frac{\cot^2 x}{\cos x} + (\operatorname{cosec}^2 x - 1) dx \\
 &= \int \frac{\cot^2 x}{\cos x} + \operatorname{cosec}^2 x - 1 dx = \int \frac{\cot^2 x}{\cos x} + \operatorname{cosec}^2 x - 1 dx \\
 &= \int \cot x \operatorname{cosec} x + \operatorname{cosec}^2 x - 1 dx = -\operatorname{cosec} x - \cot x - x + C // \\
 6. \int 1 - \sin^2 x dx &= \int 1 - (\operatorname{cosec}^2 x - 1) dx = \int 2 - \operatorname{cosec}^2 x dx \\
 &= 2x + \cot x + C //
 \end{aligned}$$

$$\begin{aligned}
 7. \int (\cos x - 3)^2 dx &= \int 4 \cos^2 x - 12 \cos x + 9 dx \\
 &= \int 4 \left(\frac{1 + \cos 2x}{2} \right) - 12 \cos x + 9 dx = \int 11 + 2 \cos 2x - 12 \cos x dx \\
 &= 11x + \sin 2x - 12 \sin x + C // \\
 8. \int (3 \sin x - \cos x)^2 dx &= \int 9 \sin^2 x - 6 \sin x \cos x + \cos^2 x dx \\
 &= \int 9 \left(\frac{1 - \cos 2x}{2} \right) - 3(2 \sin x \cos x) + \left(\frac{1 + \cos 2x}{2} \right) dx \\
 &= \int 5 - 4 \cos 2x - 3 \sin 2x dx \\
 &= 5x - 2 \sin 2x + \frac{3}{2} \cos 2x + C // \\
 9. \int \frac{1}{\sec x \sin x} dx &= \int \frac{\operatorname{cosec}^2 x}{\cos x} dx = \int \frac{1 + \cot^2 x}{\cos x} dx = \int \frac{1}{\cos x} + \frac{\cot^2 x}{\cos x} dx \\
 &= \int \sec x + \frac{\cot^2 x}{\sin x} \times \frac{1}{\cos x} dx = \int \sec x + \frac{\cot^2 x}{\sin x \cos x} dx \\
 &= \int \sec x + \frac{\cot^2 x}{\sin x} \times \frac{1}{\sin x} dx = \int \sec x + \cot x \operatorname{cosec} x dx \\
 &= \ln |\sec x + \tan x| - \operatorname{cosec} x + C // \\
 10. \int \sin x \operatorname{sech} x dx &= \int \sin x \times \frac{1}{\cosh x} dx = \int \frac{\sin x}{\cosh x} dx \\
 &= \int \operatorname{sech}^2 x - 1 dx = \tan x - x + C //
 \end{aligned}$$

Question 5

Carry out the following integrations:

$$1. \int \sin 3x \cos 2x \, dx = -\frac{1}{2} \cos x - \frac{1}{10} \cos 5x + C$$

$$2. \int \frac{1}{\sin x \cos x} \, dx = -\frac{1}{2} \ln |\operatorname{cosec} 2x + \cot 2x| + C = \ln |\tan x| + C$$

$$3. \int \frac{1}{1 - \sin x} \, dx = \sec x + \tan x + C$$

$$4. \int \sin^2 2x \, dx = \frac{1}{2} x - \frac{1}{8} \sin 4x + C$$

$$5. \int \frac{\cos 2x}{\cos^2 x} \, dx = 2x - \tan x + C$$

$$6. \int \cos^2 x \sin^2 x \, dx = \frac{1}{8} x - \frac{1}{32} \sin 4x + C$$

$$7. \int (\sin x + 2 \cos x)^2 \, dx = \frac{5}{2} x + 2 \sin^2 x + \frac{3}{4} \sin 2x + C$$

$$8. \int \frac{1}{\sin^2 x \cos^2 x} \, dx = -2 \cot 2x + C$$

$$9. \int \sqrt{\sin^2 x + (\cos x - 1)^2} \, dx = -4 \cos\left(\frac{x}{2}\right) + C$$

$$10. \int \frac{1 - \cos x}{1 + \cos x} \, dx = 2 \tan\left(\frac{x}{2}\right) - x + C = -2 \cot x - x + 2 \operatorname{cosec} x + C$$

