1 $f(x) = |2x + 3| - 4$, $x \in \mathbb{R}$

a Sketch the graph of $y = f(x)$, labelling its vertex and any points of intersection with the coordinate axes. **(5 marks)**

b Find the coordinates of the points of intersection of

$y = |2x + 3| - 4$ and $y = -\frac{1}{4}x + 2$ **(5 marks)**

2 The functions p and q are defined by $p : x \to x^2$ and $q : x \to 5 - 2x$

a Given that pq(x) = qp(x), show that $3x^2 - 10x + 10 = 0$ **(4 marks)**

b Explain why $3x^2 - 10x + 10 = 0$ has no real solutions. **(2 marks)**

3 The functions f and g are defined by $f(x) = e^{2x} + 4$, $x \in \mathbb{R}$ and $g(x) = \ln(x + 1)$, $x \in \mathbb{R}$, $x > -1$

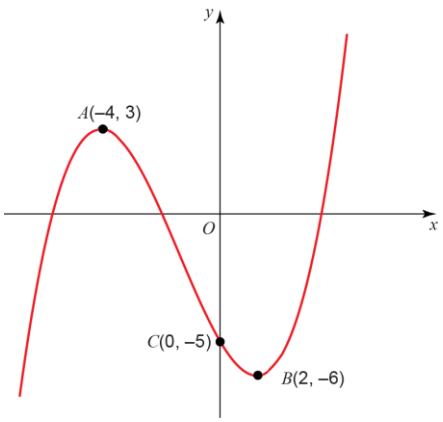
a Find fg(x) and state its range. **(4 marks)**

b Solve fg(x) = 85 **(3 marks)**

4 The function g(x) is defined by $g(x) = x^2 - 8x + 7$, $x \in \mathbb{R}$, $x > 4$. Find $g^{-1}(x)$ and state its domain and range. **(6 marks)**

5 The diagram shows the graph of h(x).

Figure 1



The points $A(-4, 3)$ and $B(2, -6)$ are turning points on the graph and $C(0, -5)$ is the y-intercept. Sketch on separate diagrams, the graphs of

a $y = |f(x)|$ **(3 marks)**

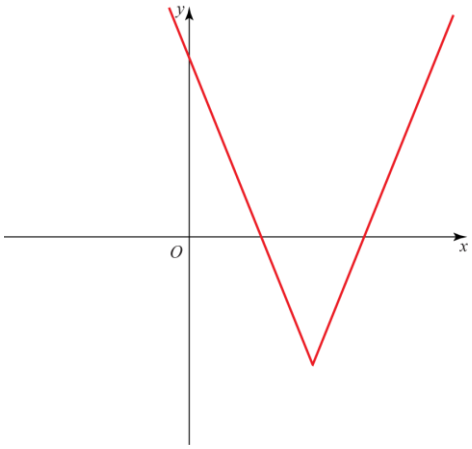
b $y = f(|x|)$ **(3 marks)**

c $y = 2f(x + 3)$ **(3 marks)**

Where possible, label clearly the transformations of the points A, B and C on your new diagrams and give their coordinates.

6 The diagram shows a sketch of part of the graph $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = 3|x - 4| - 5$

Figure 2



a State the range of f. **(1 mark)**

b Given that $\mathrm{f}(x) = -\frac{1}{3}x + k$, where k is a constant has two distinct roots, state the possible values of k. **(7 marks)**

7 The temperature of a mug of coffee at time t can be modelled by the equation $\mathrm{T}(t) = T_R + (90 - T_R)\mathrm{e}^{-\frac{1}{20}t}$, where $\mathrm{T}(t)$ is the temperature, in °C, of the coffee at time t minutes after the coffee was poured into the mug and T_R is the room temperature in °C.

a Using the equation for this model, explain why the initial temperature of the coffee is independent of the initial room temperature. **(2 marks)**

b Calculate the temperature of the coffee after 10 minutes if the room temperature is 20 °C. **(2 marks)**