

Exercise 2D

- 1** The following curves are given parametrically. In each case, find an expression for $y \frac{dx}{dt}$ in terms of t .

a $x = t + 3, y = 4t - 3$

b $x = t^3 + 3t, y = t^2$

c $x = (2t - 3)^2, y = 1 - t^2$

d $x = 6 - \frac{1}{t}, y = 4t^3, t > 0$

e $x = \sqrt{t}, y = 6t^3, t \geq 0$

f $x = \frac{4}{t^2}, y = 5t^2, t < 0$

g $x = 5t^{\frac{1}{2}}, y = 4t^{-\frac{3}{2}}, t > 0$

h $x = t^{\frac{1}{3}} - 1, y = \sqrt{t}, t \geq 0$

i $x = 16 - t^4, y = 3 - \frac{2}{t}, t < 0$

j $x = 6t^{\frac{2}{3}}, y = t^2$

- 2** A curve has parametric equations $x = 2t - 5, y = 3t + 8$. Work out $\int_0^4 y \frac{dx}{dt} dt$.
- 3** A curve has parametric equations $x = t^2 - 3t + 1, y = 4t^2$. Work out $\int_{-1}^5 y \frac{dx}{dt} dt$.
- 4** A curve has parametric equations $x = 3t^2, y = \frac{1}{t} + t^3, t > 0$. Work out $\int_{0.5}^3 y \frac{dx}{dt} dt$.
- 5** A curve has parametric equations $x = t^3 - 4t, y = t^2 - 1$. Work out $\int_{-2}^2 y \frac{dx}{dt} dt$.
- 6** A curve has parametric equations $x = 9t^{\frac{4}{3}}, y = t^{-\frac{1}{3}}, t > 0$.
- a** Show that $y \frac{dx}{dt} = a$, where a is a constant to be found.
- b** Work out $\int_3^5 y \frac{dx}{dt} dt$.
- 7** A curve has parametric equations $x = \sqrt{t}, y = 4\sqrt{t^3}, t > 0$.
- a** Show that $y \frac{dx}{dt} = pt$, where p is a constant to be found.
- b** Work out $\int_1^6 y \frac{dx}{dt} dt$.

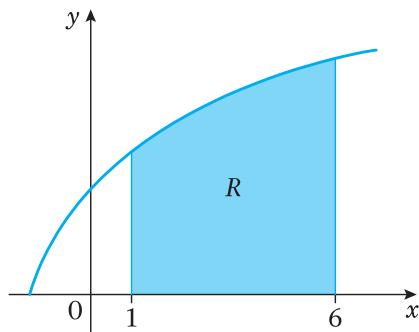
- 8** The diagram shows a sketch of the curve with parametric equations $x = t^2 - 3, y = 3t, t > 0$. The shaded region R is bounded by the curve, the x -axis and the lines $x = 1$ and $x = 6$.

a Find the value of t when

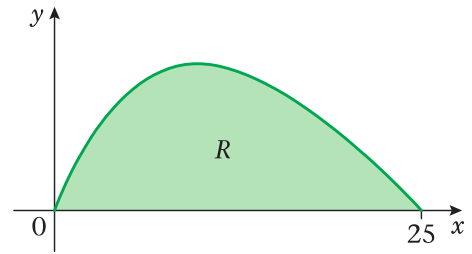
i $x = 1$

ii $x = 6$

b Find the area of R .



- 9** The diagram shows a sketch of the curve with parametric equations $x = 4t^2$, $y = t(5 - 2t)$, $t \geq 0$. The shaded region R is bounded by the curve and the x -axis. Find the area of R .



- 10** The region R is bounded by the curve with parametric equations $x = t^3$, $y = \frac{1}{3t^2}$, the x -axis and the lines $x = -1$ and $x = -8$.

a Find the value of t when

i $x = -1$ **ii** $x = -8$

b Find the area of R .

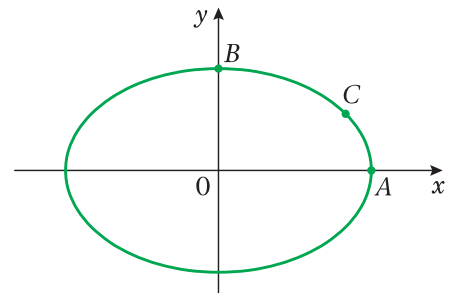
Mixed exercise 2E

- 1** The diagram shows a sketch of the curve with parametric equations $x = 4 \cos t$, $y = 3 \sin t$, $0 \leq t < 2\pi$.

a Find the coordinates of the points A and B .

b The point C has parameter $t = \frac{\pi}{6}$. Find the exact coordinates of C .

c Find the Cartesian equation of the curve.

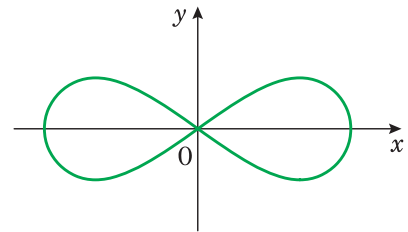


- 2** The diagram shows a sketch of the curve with parametric equations $x = \cos t$, $y = \frac{1}{2} \sin 2t$.

$0 \leq t < 2\pi$. The curve is symmetrical about both axes.

a Copy the diagram and label the points having parameters $t = 0$, $t = \frac{\pi}{2}$, $t = \pi$ and $t = \frac{3\pi}{2}$.

b Show that the Cartesian equation of the curve is $y^2 = x^2(1 - x^2)$.



- 3** A curve has parametric equations $x = \sin t$, $y = \cos 2t$, $0 \leq t < 2\pi$.

a Find the Cartesian equation of the curve.

The curve cuts the x -axis at $(a, 0)$ and $(b, 0)$.

b Find the value of a and b .

- 4** A curve has parametric equations $x = \frac{1}{1+t}$, $y = \frac{1}{(1+t)(1-t)}$, $t \neq \pm 1$.

Express t in terms of x . Hence show that the Cartesian equation of the curve is

$$y = \frac{x^2}{2x - 1}$$