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Question	Scheme	Marks	AOs
Q1	Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1 - \frac{(4\theta)^2}{2}$ in $\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$	M1	1.1b
	Attempts both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1 - \frac{(4\theta)^2}{2} \rightarrow \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$ and attempts to simplify	M1	2.1
	$= \frac{4}{3}$ oe	A1	1.1b
		(3)	
			(3 marks)

Question	Scheme	Marks	AOs
Q2 (a)	$D = 5 + 2\sin(30 \times 6.5)° = \text{awrt } 4.48\text{m}$ with units	B1	3.4
		(1)	
(b)	$3.8 = 5 + 2\sin(30t)° \Rightarrow \sin(30t)° = -0.6$	M1 A1	1.1b 1.1b
	$t = 10.77$	dM1	3.1a
	10:46 a.m. or 10:47 a.m.	A1	3.2a
		(4)	
			(5 marks)

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Question	Scheme	Marks	AOs
Q3	(i) $4\sin x = \sec x,\ 0 \le x < \frac{\pi}{2}$; (ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$		
(i) Way 1	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 2\sin 2x = 1 \Rightarrow \sin 2x = \frac{1}{2}$	M1	3.1a
	$x = \frac{1}{2}\arcsin\left(\frac{1}{2}\right)$ or $\frac{1}{2}\left(\pi - \arcsin\left(\frac{1}{2}\right)\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	
(i) Way 2	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 16\sin^2 x\cos^2 x = 1$ $16\sin^2 x(1-\sin^2 x) = 1$ | $16(1-\cos^2 x)\cos^2 x = 1$ $16\sin^4 x - 16\sin^2 x + 1 = 0$ | $16\cos^4 x - 16\cos^2 x + 1 = 0$ $\sin^2 x$ or $\cos^2 x = \frac{16 \pm \sqrt{192}}{32}\ \left\{= \frac{2\pm\sqrt{3}}{4} \text{ or } 0.933..., 0.066...\right\}$	M1	3.1a
	$x = \arcsin\left(\sqrt{\frac{2\pm\sqrt{3}}{4}}\right)$ or $x = \arccos\left(\sqrt{\frac{2\pm\sqrt{3}}{4}}\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	
(ii)	Complete strategy, i.e. • Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\sin(\theta - \alpha) = 2$, finds both R and α, and proceeds to $\sin(\theta-\alpha) = k,\ \lvert k\rvert < 1,\ k \ne 0$ • Applies $(5\sin\theta - 5\cos\theta)^2 = 2^2$, followed by applying both $\cos^2\theta + \sin^2\theta = 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to proceed to $\sin 2\theta = k,\ \lvert k\rvert < 1,\ k \ne 0$	M1	3.1a
	$R = \sqrt{50}$ $\tan\alpha = 1 \Rightarrow \alpha = 45°$ | $(5\sin\theta - 5\cos\theta)^2 = 2^2 \Rightarrow$ $25\sin^2\theta + 25\cos^2\theta - 50\sin\theta\cos\theta = 4$ $\Rightarrow 25 - 25\sin 2\theta = 4$	M1	1.1b
	$\sin(\theta - 45°) = \frac{2}{\sqrt{50}}$ | $\sin 2\theta = \frac{21}{25}$	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arcsin\left(\frac{2}{\sqrt{50}}\right) + 45°$ | e.g. $\theta = \frac{1}{2}\left(\arcsin\left(\frac{21}{25}\right)\right)$	dM1	1.1b
	$\theta =$ awrt $61.4°$, awrt $208.6°$	A1	2.1
	Note: Working in radians does not affect any of the first 4 marks		
		(5)	
			(9 marks)

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Question	Scheme		Marks	AOs
Q3	(ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$			
(ii) **Alt 1**	Complete strategy, i.e. • Attempts to apply $(5\sin\theta)^2 = (2+5\cos\theta)^2$ or $(5\sin\theta-2)^2 = (5\cos\theta)^2$ followed by applying $\cos^2\theta+\sin^2\theta=1$ and solving a quadratic equation in either $\sin\theta$ or $\cos\theta$ to give at least one of $\sin\theta = k$ or $\cos\theta = k,\ \lvert k\rvert<1,\ k\ne 0$		M1	3.1a
	e.g. $25\sin^2\theta = 4+20\cos\theta+25\cos^2\theta$ $\Rightarrow 25(1-\cos^2\theta) = 4+20\cos\theta+25\cos^2\theta$ **or** e.g. $25\sin^2\theta-20\sin\theta+4 = 25\cos^2\theta$ $\Rightarrow 25\sin^2\theta-20\sin\theta+4 = 25(1-\sin^2\theta)$		M1	1.1b
	$50\cos^2\theta+20\cos\theta-21=0$	$50\sin^2\theta-20\sin\theta-21=0$		
	$\cos\theta = \dfrac{-20\pm\sqrt{4600}}{100}$, o.e.	$\sin\theta = \dfrac{20\pm\sqrt{4600}}{100}$, o.e.	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arccos\left(\dfrac{-2+\sqrt{46}}{10}\right)$	e.g. $\theta = \arcsin\left(\dfrac{2+\sqrt{46}}{10}\right)$	dM1	1.1b
	$\theta =$ awrt $61.4°$, awrt $208.6°$		A1	2.1
			(5)	

Q4

$\cos 3\theta = \cos(2\theta+\theta)$

$= \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$ B1

$= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta$ B1

$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta(1-\cos^2\theta)$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$ A1

$= 4\cos^3\theta - 3\cos\theta$

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Question Number	Scheme	Marks
Q5 (i)	$\dfrac{\tan 2x+\tan 32°}{1-\tan 2x\tan 32°}=5\Rightarrow \tan(2x+32°)=5$	B1
	$\Rightarrow x=\dfrac{\arctan 5-32°}{2}$	M1
	$\Rightarrow x=\text{awrt } 23.35°, -66.65°$	A1A1
		(4)
(ii)(a)	$\tan(3\theta-45°)=\dfrac{\tan 3\theta-\tan 45°}{1+\tan 45°\tan 3\theta}=\dfrac{\tan 3\theta-1}{1+\tan 3\theta}$	M1A1*
		(2)
(b)	$(1+\tan 3\theta)\tan(\theta+28°)=\tan 3\theta-1$	
	$\Rightarrow \tan(\theta+28°)=\tan(3\theta-45°)$	B1
	$\theta+28°=3\theta-45°\Rightarrow\theta=36.5°$	M1A1
	$\theta+28°+180°=3\theta-45°\Rightarrow\theta=126.5°$	dM1A1
		(5)
		(11 marks)
Q5 **(i)** **ALT 1**	$\dfrac{\tan 2x+\tan 32°}{1-\tan 2x\tan 32°}=5\Rightarrow \tan 2x=\dfrac{5-\tan 32°}{1+5\tan 32°}=awrt 1.06$	B1
	$\Rightarrow x=\dfrac{\arctan\left(\dfrac{5-\tan 32°}{1+5\tan 32°}\right)}{2}$	M1
	$\Rightarrow x=23.35°, -66.65°$	A1A1
		(4)
Q5 **(ii)** **ALT 2**	$\dfrac{\tan 2x+\tan 32°}{1-\tan 2x\tan 32°}=5\Rightarrow\dfrac{2\tan x}{1-\tan^2 x}+\tan 32°=5-5\times\dfrac{2\tan x}{1-\tan^2 x}\tan 32°$	
	$\Rightarrow(5-\tan 32°)\tan^2 x+(2+10\tan 32°)\tan x+\tan 32°-5=0$	
	OR $\Rightarrow awrt\ 4.38\tan^2 x+8.25\tan x-4.38=0$	B1
	Quadratic formula $\Rightarrow \tan x=0.4316, -2.3169\Rightarrow x=..$	M1
	$\Rightarrow x=23.35°, -66.65°$	A1 A1
		(4)

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Question Number	Scheme	Marks
Q6 **(a)**	$R=\sqrt{5}$	B1
	$\tan\alpha = 2 \Rightarrow \alpha = \text{awrt } 1.107$	M1A1
		(3)
(b)(i)	$'40+9R^2' = 85$	M1A1
(ii)	$\theta = \frac{\pi}{2} + 1.107 \Rightarrow \theta = \text{awrt } 2.68$	B1ft
		(3)
(c)(i)	6	B1
(ii)	$2\theta - '1.107' = 3\pi \Rightarrow \theta = \text{awrt } 5.27$	M1A1
		(3)
		(9 marks)

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Q7	Scheme	Notes	Marks
	$\cot x - \tan x \equiv 2\cot 2x$		
(a)	$\cot x - \tan x = \dfrac{\cos x}{\sin x} - \dfrac{\sin x}{\cos x}$	Attempts to write both $\cot x$ and $\tan x$ in terms of $\sin x$ and $\cos x$ only	M1
	$= \dfrac{\cos^2 x}{\sin x \cos x} - \dfrac{\sin^2 x}{\cos x \sin x}\left(= \dfrac{\cos^2 x - \sin^2 x}{\sin x \cos x}\right)$	**Dependent on the previous M mark** Attempts to find the same denominator for both fractions	dM1
	$= \dfrac{\cos 2x}{\frac{1}{2}\sin 2x}\left(= \dfrac{2\cos 2x}{\sin 2x}\right)$	**Dependent on both the previous M marks.** Evidence of correctly applying either $\cos 2x = \cos^2 x - \sin^2 x$ or $\sin 2x = 2\sin x \cos x$	ddM1
	$= 2\cot 2x$ (*)	Correct proof with no notational or other errors such as missing x's or inconsistent variables.	A1 *
			[4]
(a) **Alt 1**	$\cot x - \tan x = \dfrac{1}{\tan x} - \tan x$	Writes $\cot x$ in terms of $\tan x$	M1
	$\dfrac{1}{\tan x} - \dfrac{\tan^2 x}{\tan x}\left(= \dfrac{1 - \tan^2 x}{\tan x}\right)$	**Dependent on the previous M mark** Attempts to find the same denominator for both fractions	dM1
	$\dfrac{2}{\tan 2x}$	**Dependent on both the previous M marks.** Evidence of correctly applying $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	ddM1
	$= 2\cot 2x$ (*)	Correct proof with no notational or other errors such as missing x's or inconsistent variables.	A1*
			[4]
(a) **Alt 2**	$2\cot 2x = \dfrac{2}{\tan 2x}$	Applies $\cot 2x = \dfrac{1}{\tan 2x}$	M1
	$= \dfrac{2}{\frac{2\tan x}{1 - \tan^2 x}}$	**Dependent on the previous M mark** Attempts to apply the double angle formula for $\tan 2x$	dM1
	$= \dfrac{1 - \tan^2 x}{\tan x} = \dfrac{1}{\tan x} - \tan x$	**Dependent on both the previous M marks.** Obtains a rational fraction with a single denominator and attempts to split this up into 2 terms	ddM1
	$= \cot x - \tan x$ (*)	Correct proof with no notational or other errors such as missing x's or inconsistent variables.	A1 *
			[4]

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(b)	$5 + \cot(\theta - 15°) - \tan(\theta - 15°) = 0$		
	$\Rightarrow 5 + 2\cot(...) = 0$	Obtains an equation of this form.	M1
	$\cot(...) = -\frac{5}{2} \Rightarrow \tan(...) = -\frac{2}{5}$	Obtains an equation of the form $\tan(...) = \pm\frac{2}{5}$	M1
	$2\theta - 30 = \tan^{-1}\left(-\frac{2}{5}\right)$	Can be implied by e.g. $2\theta - 30 =$ awrt -21.8 or $2\theta - 30 =$ awrt 158.2	A1
	$\theta =$ awrt 4.1° **or** $\theta =$ awrt 94.1°	One correct answer e.g. anything that rounds to 4.1 or anything that rounds to 94.1	A1
	$\theta =$ awrt 4.1° **and** $\theta =$ awrt 94.1°	Both answers correct. Ignore any extra answers out of range but withhold this mark if there are any extra values in range.	A1
			[5]
	Alternative to part (b):		
	$5 + \cot(...) - \tan(...) = 0 \Rightarrow 5\tan(...) + 1 - \tan^2(...)$ $\tan^2(...) - 5\tan(...) - 1 = 0$ Multiples through by $\tan(...)$ to obtain a 3TQ in $\tan(...)$		M1
	$\tan(...) = \frac{5 \pm \sqrt{25+4}}{2}$	Solves their 3TQ and proceeds to tan(…) =	M1
	$(\theta - 15°) = \tan^{-1}\left(\frac{5 \pm \sqrt{25+4}}{2}\right)$	Can be implied by e.g. $\theta - 15 = 79.099...$ or $\theta - 15 = -10.900...$	A1
	$\theta =$ awrt 4.1° **or** $\theta =$ awrt 94.1°	One correct answer e.g. anything that rounds to 4.1 or anything that rounds to 94.1	A1
	$\theta =$ awrt 4.1° **and** $\theta =$ awrt 94.1°	Both answers correct. Ignore any extra answers out of range but withhold this mark if there are any extra values in range.	A1
			[5]
			9

		Question 12 Notes
(a)	**Note**	Allow candidates to "meet in the middle" e.g. $\text{lhs} = \frac{1}{\tan x} - \tan x = \frac{1 - \tan^2 x}{\tan x}$: M1dM1 as in Alt1 $\text{rhs} = 2\cot 2x = \frac{2}{\tan 2x} = \frac{2}{\frac{2\tan x}{1-\tan^2 x}}$: ddM1 uses double angle for tan2x on rhs $= \frac{1-\tan^2 x}{\tan x}$ so lhs = rhs A1 Correct proof with conclusion

MAX is 50, Give mark as a percentage.