Paper Reference(s)

3142

Beal GCE

A2 Level

Trigonometry Assessment

Advanced Level (A2)

November 2018

Time: 1 hour

Calculators may be used in this examination.

Instructions to Candidates
On the exercise paper provided make sure you write your name in **BLOCK CAPITALS** clearly.

Information for Candidates
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions on this paper. The total mark for this paper is 50.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labeled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may gain no credit.

Turn Over

1. Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1-\cos 4\theta}{2\theta \sin 3\theta}.$$

(Total 3 marks)

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2. The depth of water, D metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)°, \quad 0 \le t < 24,$$

where t is the number of hours after midnight.

A boat enters the harbour at 6:30 a.m. and it takes 2 hours to load its cargo. The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

(1)

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

(Total 5 marks)

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3. (i) Solve, for $0 \le x < \frac{\pi}{2}$, the equation

$$4\sin x = \sec x.$$ **(4)**

(ii) Solve, for $0 \le \theta < 360°$, the equation

$$5\sin\theta - 5\cos\theta = 2,$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.) **(5)**

(Total 9 marks)

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4. Show that

$$\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$$

(Total 4 marks)

5. (i) Using the identity for tan $(A \pm B)$, solve, for $-90° < x < 90°$,

$$\frac{\tan 2x + \tan 32°}{1 - \tan 2x \tan 32°} = 5$$

Give your answers, in degrees, to 2 decimal places.

(4)

(ii) (*a*) Using the identity for tan $(A \pm B)$, show that

$$\tan(3\theta - 45°) \equiv \frac{\tan 3\theta - 1}{1 + \tan 3\theta}, \qquad \theta \neq (60n + 45)°,\ n \in \mathbb{Z}$$

(2)

(*b*) Hence solve, for $0 < \theta < 180°$,

$$(1 + \tan 3\theta)\tan(\theta + 28°) = \tan 3\theta - 1$$

(5)

(Total 11 marks)

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6. (*a*) Express $\sin\theta - 2\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and the value of α, in radians, to 3 decimal places.

(3)

$$M(\theta) = 40 + (3\sin\theta - 6\cos\theta)^2$$

(*b*) Find

(i) the maximum value of $M(\theta)$,

(ii) the smallest value of θ, in the range $0 < \theta \leqslant 2\pi$, at which the maximum value of $M(\theta)$ occurs.

(3)

$$N(\theta) = \frac{30}{5 + 2(\sin 2\theta - 2\cos 2\theta)^2}$$

(*c*) Find

(i) the maximum value of $N(\theta)$,

(ii) the largest value of θ, in the range $0 < \theta \leqslant 2\pi$, at which the maximum value of $N(\theta)$ occurs.

(3)

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(Total 9 marks)

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7. (*a*) Show that

$$\cot x - \tan x \equiv 2 \cot 2x, \qquad x \neq 90n°, n \in \mathbb{Z}$$

(4)

(*b*) Hence, or otherwise, solve, for $0 \leqslant \theta \leqslant 180°$

$$5 + \cot(\theta - 15°) - \tan(\theta - 15°) = 0$$

giving your answers to one decimal place.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(5)

(Total 9 marks)