Paper Reference(s)

6663/01

Beal Edexcel GCE

Core Mathematics

Differentiation Assessment

Advanced Subsidiary

January 2019

Time: 1 hour

Materials required for examination	**Items included with question papers**
Mathematical Formulae	Nil

Calculators may be used in all questions.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics AS), the paper reference (6663), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.

There are 5 questions in this question paper. The total mark for this paper is 50.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. (i) If $y = 2x^2 - 4x - x^2\sqrt{x}, \quad x > 0$

(a) Find $\frac{dy}{dx}$ (3)

(b) Find the value of k such that

$$\frac{d^2y}{dx^2} + k\sqrt{x} = 4$$ (3)

(ii) The curve C has equation $y = 3x^3 - 2x^2$

Find the equation of the normal to C at $x = -1$

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. (5)

(Total 11 marks)

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2. A curve C has equation

$$y = x^2 - 2x - 24\sqrt{x}, \quad x > 0.$$

(a) Find (i) $\frac{dy}{dx}$,

(ii) $\frac{d^2y}{dx^2}$.

(3)

(b) Verify that C has a stationary point when $x = 4$.

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

(Total 7 marks)

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3. Prove, from first principles, that the derivative of $5x^2$ is $10x$

(**Total 4 marks**)

4.

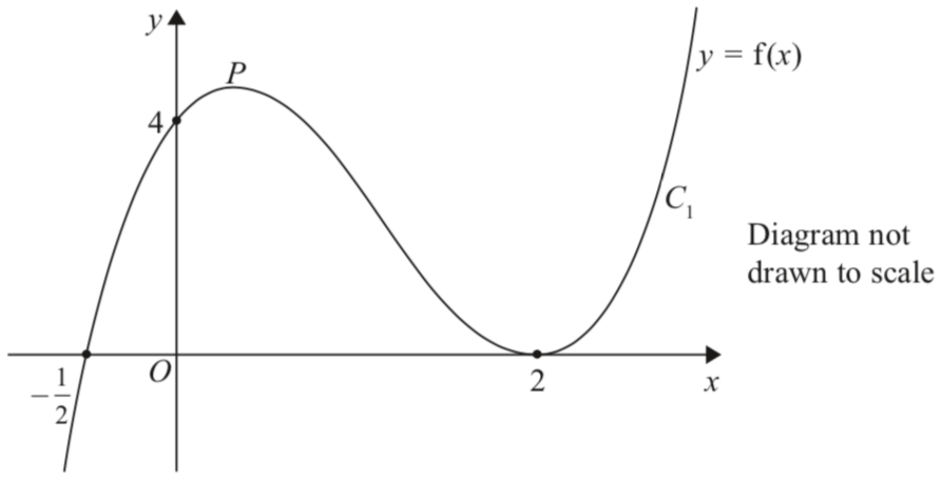

Figure 2

Figure 2 shows a sketch of the curve C_1 with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = (x - 2)^2(2x + 1), \qquad x \in \mathbb{R}$$

The curve crosses the x-axis at $\left(-\frac{1}{2}, 0\right)$, touches it at (2, 0) and crosses the y-axis at (0, 4). There is a maximum turning point at the point marked P.

(*a*) Use $\mathrm{f}'(x)$ to find the exact coordinates of the turning point P.

(7)

A second curve C_2 has equation $y = \mathrm{f}(x + 1)$.

(*b*) Write down an equation of the curve C_2
You may leave your equation in a factorised form.

(1)

(*c*) Use your answer to part (*b*) to find the coordinates of the point where the curve C_2 meets the y-axis.

(2)

(*d*) Write down the coordinates of the two turning points on the curve C_2

(2)

(*e*) Sketch the curve C_2, with equation $y = \mathrm{f}(x + 1)$, giving the coordinates of the points where the curve crosses or touches the x-axis.

(3)

(Total 15 marks)

TURN OVER

5.

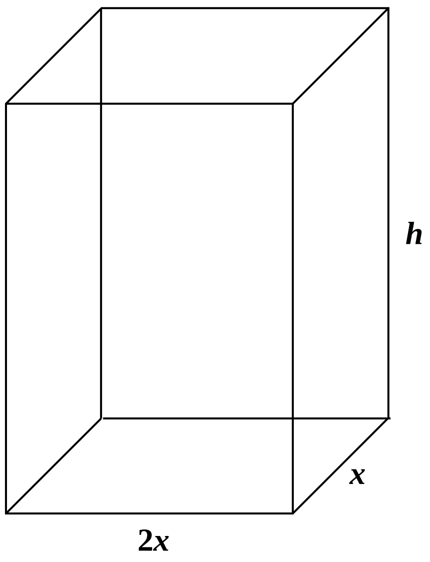

Fig. 3

A manufacturer produces cartons for fruit juice. Each carton is in the shape of a closed cuboid with base dimensions $2x$ cm by x cm and height h cm, as shown in Fig. 3.

Given that the capacity of a carton has to be 1030 cm^3, and the surface area is A cm^2.

Use calculus to find the minimum surface area A, as x varies and show that this value is a minimum.

(Total 13 marks)

END OF EXAM

Deconstructed AS Differentiation Questions

1. The curve C has equation

$$y = 2x^3 - 5x^2 - 4x + 2.$$

Find the turning points of C and determine their nature.

(10 marks)

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2. A diesel lorry is driven from Birmingham to Bury at a steady speed of v kilometres per hour.

The total cost of the journey, £C, is given by

$$C = \frac{1400}{v} + \frac{2v}{7}.$$

Calculate the minimum total cost of the journey and show that it is a minimum.

(9 marks)

3.

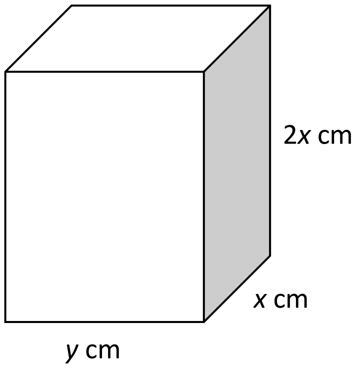

Figure 1

Figure 1 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2.

Given that x varies,

(a) use calculus to find the maximum value of V, giving your answer to the nearest cm^3.

(9)

(b) Justify that the value of V you have found is a maximum.

(2)

(11 marks)

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4. The curve C has equation $y = 12\sqrt{(x)} - x^{\frac{3}{2}} - 10,\ x > 0.$

Use calculus to find the coordinates of the turning point on C and determine the nature of this turning point.

(10 marks)

5. A solid right circular cylinder has radius r cm and height h cm.

The total surface area of the cylinder is 800 cm^2 .

Given that r varies,

(a) use calculus to find the maximum value of V, to the nearest cm^3.

(10)

(b) Justify that the value of V you have found is a maximum.

(2)
(12 marks)

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6. The curve C has equation $y = 4x^2 + \dfrac{5-x}{x}$, $x \neq 0$. The point P on C has x-coordinate 1.

If the tangent at P meets the x-axis at the point (k, 0), find the value of k.(a)

(10 marks)

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7. The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2$, $x > 0$.

$P(4, 8)$ lies on C

The normal to C at P cuts the x-axis at the point Q.

Find the length PQ, giving your answer in a simplified surd form.

(10 marks)

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8. The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \quad x > 0.$$

The point P on C has x-coordinate equal to 2.

The tangent at P meets the x-axis at A and the normal at P meets the x-axis at B.

Find the area of the triangle APB.

(13 marks)

Paper Reference(s)

6663/01

Beal Edexcel GCE

Core Mathematics

Differentiation Assessment

Advanced Subsidiary

January 2018

Time: 1 hour

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Calculators may be used in all questions.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics AS), the paper reference (6663), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 48.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. Given

$$y = \sqrt{x} + \frac{4}{\sqrt{x}} + 4, \qquad x > 0$$

find the value of $\frac{dy}{dx}$ when $x = 8$, writing your answer in the form $a\sqrt{2}$, where a is a rational number.

(Total 5 marks)

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2. Show that the normal to the curve C with equation

$$y = x^3 - 9x^2 + 26x - 18$$

at the point (4, 6) which lies on C, cuts the x-axis at (16,0)

(Total 6 marks)

3.
$$f(x) = \frac{\left(4 + 3\sqrt{x}\right)^2}{x}, \qquad x > 0$$

(*a*) Show that $f(x) = Ax^{-1} + Bx^k + C$, where A, B, C and k are constants to be determined.

(4)

(*b*) Hence find $f'(x)$.

(2)

(*c*) Find an equation of the tangent to the curve $y = f(x)$ at the point where $x = 4$

(4)

(Total 10 marks)

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4. Prove, from first principles, that the derivative of $7x^3$ is $21x^2$

(**Total 4 marks**)

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5. The curve C has equation $y = 4x\sqrt{x} + \dfrac{48}{\sqrt{x}} - \sqrt{8}, \qquad x > 0$

(a) Find, simplifying each term,

(i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(5)

(b) Use part (a) to find the exact coordinates of the stationary point of C.

(5)

(c) Determine whether the stationary point of C is a maximum or minimum, giving a reason for your answer.

(2)

(Total 12 marks)

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5. A pencil holder is in the shape of an open circular cylinder of radius r cm and height h cm.

The surface area of the cylinder (including the base) is 250 cm^2.

(a) Show that the volume, V cm^3, of the cylinder is given by $V = 125r - \dfrac{\pi r^3}{2}$.

(4)

(b) Use calculus to find the value of r for which V has a stationary value.

(3)

(c) Prove that the value of r you found in part (b) gives a maximum value for V.

(2)

(d) Calculate, to the nearest cm^3, the maximum volume of the pencil holder.

(2)

(Total 11 marks)

TOTAL FOR PAPER: 48 MARKS

END

C2 Maximum and Minimum Problems

Q1

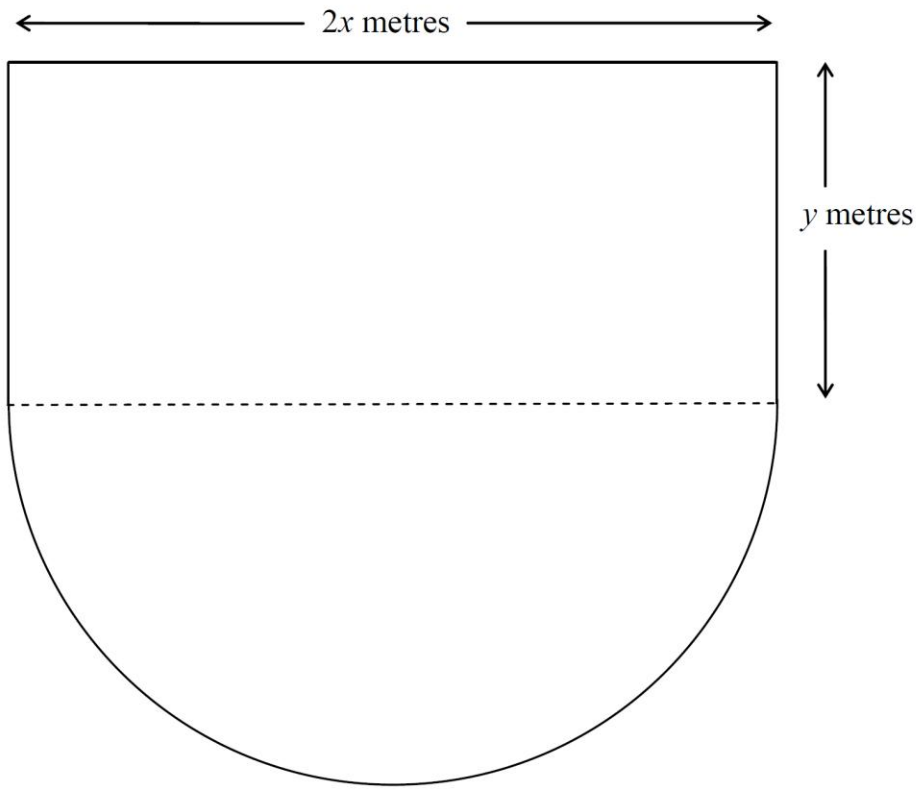

Figure 3

Figure 3 shows the plan of a stage in the shape of a rectangle joined to a semicircle. The length of the rectangular part is $2x$ metres and the width is y metres. The diameter of the semicircular part is $2x$ metres. The perimeter of the stage is 80 m.

(*a*) Show that the area, A m^2, of the stage is given by

$$A = 80x - \left(2 + \frac{\pi}{2}\right)x^2.$$

(4)

(*b*) Use calculus to find the value of x at which A has a stationary value.

(4)

(*c*) Prove that the value of x you found in part (*b*) gives the maximum value of A.

(2)

(*d*) Calculate, to the nearest m^2, the maximum area of the stage.

(2)

Q2

A diesel lorry is driven from Birmingham to Bury at a steady speed of v kilometres per hour. The total cost of the journey, £C, is given by

$$C = \frac{1400}{v} + \frac{2v}{7}.$$

(a) Find the value of v for which C is a minimum.

(5)

(b) Find $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$ and hence verify that C is a minimum for this value of v.

(2)

(c) Calculate the minimum total cost of the journey.

(2)

Q3

A solid right circular cylinder has radius r cm and height h cm.

The total surface area of the cylinder is 800 cm^2 .

(a) Show that the volume, V cm^3 , of the cylinder is given by

$$V = 400r - \pi r^3.$$

(4)

Given that r varies,

(b) use calculus to find the maximum value of V, to the nearest cm^3.

(6)

(c) Justify that the value of V you have found is a maximum.

(2)

Q4

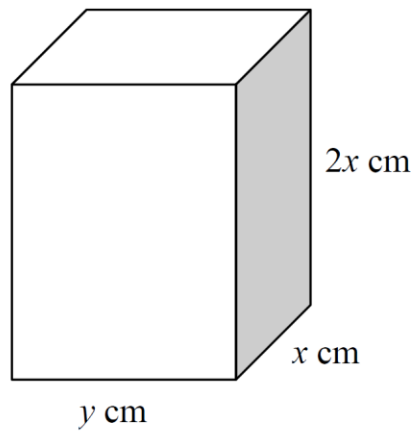

Figure 4

Figure 4 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2.

(*a*) Show that the volume, V cm^3, of the brick is given by

$$V = 200x - \frac{4x^3}{3}.$$

(4)

Given that x can vary,

(*b*) use calculus to find the maximum value of V, giving your answer to the nearest cm^3.

(5)

(*c*) Justify that the value of V you have found is a maximum.

(2)

Q5

Figure 4

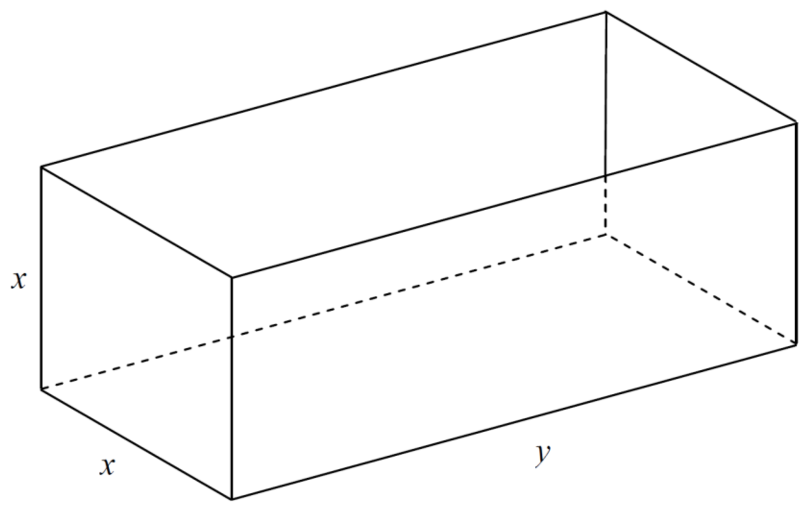

Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is 100 m^3.

(*a*) Show that the area $A \text{ m}^2$ of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2.$$

(4)

(*b*) Use calculus to find the value of x for which A is stationary.

(4)

(*c*) Prove that this value of x gives a minimum value of A.

(2)

(*d*) Calculate the minimum area of sheet metal needed to make the tank.

(2)

END