Mixed Trigonometry Questions

Pythagoras Type

1 Given that angle A is obtuse and $\cos A = -\sqrt{\frac{7}{11}}$, show that $\tan A = \frac{-2\sqrt{7}}{7}$.

2 Given that angle B is reflex and $\tan B = +\frac{\sqrt{21}}{2}$, find the exact value of: **a** $\sin B$, **b** $\cos B$.

Trigonometric Identity

4 Simplify the following expressions:

a $\cos^4\theta - \sin^4\theta$ **b** $\sin^2 3\theta - \sin^2 3\theta\cos^2 3\theta$

c $\cos^4\theta + 2\sin^2\theta\cos^2\theta + \sin^4\theta$

5 **a** Given that $2(\sin x + 2\cos x) = \sin x + 5\cos x$, find the exact value of $\tan x$.

b Given that $\sin x\cos y + 3\cos x\sin y = 2\sin x\sin y - 4\cos x\cos y$, express $\tan y$ in terms of $\tan x$.

6 Show that, for all values of θ:

a $(1 + \sin\theta)^2 + \cos^2\theta = 2(1 + \sin\theta)$ **b** $\cos^4\theta + \sin^2\theta = \sin^4\theta + \cos^2\theta$

Graphs Problem Solving

3 **a** Sketch the graph of $y = \sin(x + 60)°$, in the interval $-360 \leqslant x \leqslant 360$, giving the coordinates of points of intersection with the axes.

b Calculate the values of the x-coordinates of the points in which the line $y = \frac{1}{2}$ intersects the curve.

7 Without attempting to solving them, state how many solutions the following equations have in the interval $0 \leqslant \theta \leqslant 360°$. Give a brief reason for your answer.

a $2\sin\theta = 3$

b $\sin\theta = .-\cos\theta$

c $2\sin\theta + 3\cos\theta + 6 = 0$

d $\tan\theta + \dfrac{1}{\tan\theta} = 0$

18 Consider the function f(x) defined by

$\text{f}(x) \equiv 3 + 2\sin(2x + k)°,\ 0 < x < 360$

where k is a constant and $0 < k < 360$. The curve with equation $y = \text{f}(x)$ passes through the point with coordinates $(15, 3 + \sqrt{3})$.

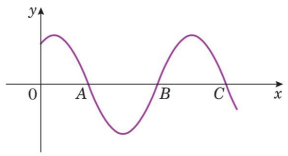
a Show that $k = 30$ is a possible value for k and find the other possible value of k.

b Given that $k = 30$, solve the equation $\text{f}(x) = 1$.

E

19 **a** Determine the solutions of the equation $\cos(2x - 30)° = 0$ for which $0 \leqslant x \leqslant 360$.

b The diagram shows part of the curve with equation $y = \cos(px - q)°$, where p and q are positive constants and $q < 180$. The curve cuts the x-axis at points A, B and C, as shown.



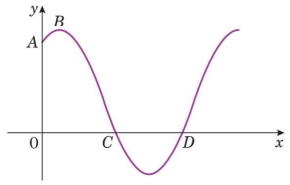
Given that the coordinates of A and B are (100, 0) and (220, 0) respectively:

i write down the coordinates of C,

ii find the value of p and the value of q.

E

20 The diagram shows part of the curve with equation $y = \text{f}(x)$, where $\text{f}(x) = 1 + 2\sin(px° + q°)$, p and q being positive constants and $q \leqslant 90$. The curve cuts the y-axis at the point A and the x-axis at the points C and D. The point B is a maximum point on the curve.



Given that the coordinates of A and C are (0, 2) and (45, 0) respectively:

a calculate the value of q,

b show that $p = 4$,

c find the coordinates of B and D.

E

Equations

1 Solve for θ, in the interval $0 \leqslant \theta \leqslant 360°$, the following equations.
Give your answers to 3 significant figures where they are not exact.

a $4\cos^2\theta = 1$

b $2\sin^2\theta - 1 = 0$

c $3\sin^2\theta + \sin\theta = 0$

d $\tan^2\theta - 2\tan\theta - 10 = 0$

e $2\cos^2\theta - 5\cos\theta + 2 = 0$

f $\sin^2\theta - 2\sin\theta - 1 = 0$

g $\tan^2 2\theta = 3$

h $4\sin\theta = \tan\theta$

i $\sin\theta + 2\cos^2\theta + 1 = 0$

j $\tan^2(\theta - 45°) = 1$

k $3\sin^2\theta = \sin\theta\cos\theta$

l $4\cos\theta(\cos\theta - 1) = -5\cos\theta$

m $4(\sin^2\theta - \cos\theta) = 3 - 2\cos\theta$

n $2\sin^2\theta = 3(1 - \cos\theta)$

o $4\cos^2\theta - 5\sin\theta - 5 = 0$

p $\cos^2\dfrac{\theta}{2} = 1 + \sin\dfrac{\theta}{2}$

2 Solve for θ, in the interval $-180° \leqslant \theta \leqslant 180°$, the following equations.
Give your answers to 3 significant figures where they are not exact.

a $\sin^2 2\theta = 1$

b $\tan^2\theta = 2\tan\theta$

c $\cos\theta(\cos\theta - 2) = 1$

d $\sin^2(\theta + 10°) = 0.8$

e $\cos^2 3\theta - \cos 3\theta = 2$

f $5\sin^2\theta = 4\cos^2\theta$

g $\tan\theta = \cos\theta$

h $2\sin^2\theta + 3\cos\theta = 1$

3 Solve for x, in the interval $0 \leqslant x \leqslant 2\pi$, the following equations.
Give your answers to 3 significant figures unless they can be written in the form $\dfrac{a}{b}\pi$, where a and b are integers.

a $\tan^2\frac{1}{2}x = 1$

b $2\sin^2\left(x + \dfrac{\pi}{3}\right) = 1$

c $3\tan x = 2\tan^2 x$

d $\sin^2 x + 2\sin x\cos x = 0$

e $6\sin^2 x + \cos x - 4 = 0$

f $\cos^2 x - 6\sin x = 5$

g $2\sin^2 x = 3\sin x\cos x + 2\cos^2 x$